

# A Knee Solution-Based Membrane-Inspired Evolutionary Algorithm for Multi-objective Multi-task Optimization

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**Abstract.** A novel knee solution-based membrane-inspired evolutionary algorithm (Knee-MOMTMIEA) is proposed to solve multi-objective multi-task optimization problems. The algorithm integrates hierarchical membrane structures with a knee solution-based information transfer mechanism to enable efficient and adaptive knowledge sharing among tasks. By utilizing knee solutions as representative individuals, the approach enhances convergence and solution quality. Comprehensive experiments on the classical MOMTO test suite validate the algorithm’s effectiveness, demonstrating that Knee-MOMTMIEA consistently outperforms state-of-the-art multi-task optimization algorithms. This work represents a significant advancement in integrating membrane computing with evolutionary multitasking, offering an efficient framework for solving MOMTO problems.

**Keywords:** Multi-objective multi-task optimization · Membrane computing · Evolutionary multitasking · Knee solutions.

## 1 Introduction

In numerous scientific and engineering domains, it is often necessary to optimize several conflicting objectives simultaneously. Such challenges are categorized as multi-objective optimization problems (MOPs). To generalize these problems, and assuming that all objectives are to be minimized, a MOP can be mathematically formulated as:

$$\min_{\mathbf{x} \in \Omega} F(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_M(\mathbf{x})] \quad (1)$$

In this expression,  $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_D)$  represents a decision vector within a  $D$ -dimensional space,  $\Omega$  denotes the feasible region in the decision space, and  $F(x)$  is the vector comprising  $M$  objective functions. Due to the conflicting nature of the objectives, it is impossible to identify a single solution that simultaneously optimizes all objective

functions. To address this issue, the concept of Pareto optimality is introduced, aiming to find solutions that represent the best possible trade-offs among the objectives. Specifically, given two decision vectors  $\mathbf{x}$  and  $\mathbf{y}$ ,  $\mathbf{x}$  is said to Pareto dominate  $\mathbf{y}$ , (denoted as  $\mathbf{x} \prec \mathbf{y}$ ) if it satisfies two conditions: first,  $f_i(\mathbf{x}) \leq f_i(\mathbf{y})$  for every objective  $i \in \{1, 2, \dots, M\}$ ; second, there exists at least one objective  $j$  such that  $f_j(\mathbf{x}) < f_j(\mathbf{y})$ . A solution  $\mathbf{x}^*$  is considered a Pareto optimal solution if no other solution in the feasible region dominates it according to the Pareto dominance criteria. The collection of all such non-dominated solutions forms the Pareto optimal set (PS), and its corresponding representation in the objective space is known as the Pareto optimal front (PF) [1].

Multi-objective evolutionary algorithms (MOEAs) have become indispensable for addressing multi-objective optimization problems (MOPs) due to their ability to efficiently approximate the Pareto front with high convergence and diversity in a single run. Typically, MOEAs are categorized into three main groups: those based on Pareto dominance [2], those utilizing performance indicators [3], and those employing decomposition methods [4]. While these algorithms are effective, they are primarily designed to solve one MOP at a time, requiring population reinitialization when encountering new problems. However, in practical applications, many MOPs exhibit interconnections where leveraging knowledge acquired from solving one problem can positively impact the optimization of related tasks by transferring valuable information and accelerating convergence.

To enhance the generalization performance of evolutionary algorithms, the concept of multi-task optimization (MTO) has been proposed [5]. Inspired by multi-task learning and transfer learning in machine learning, MTO aims to exploit relationships among multiple optimization tasks by solving them simultaneously and leveraging shared knowledge to improve overall performance. Unlike traditional multi-objective evolutionary algorithms (MOEAs), which address one problem at a time, evolutionary multitasking (EMT) algorithms aim to find optimal solutions for all tasks concurrently by utilizing correlations between them. This strategy not only accelerates the search process but also enhances solution quality. Due to its powerful search capabilities and innovative theoretical perspectives, MTO has become a prominent research focus in the field of evolutionary computation [6].

Membrane computing, also known as P system, a branch of natural computing, draws inspiration from the structural and functional characteristics of biological membranes within living cells [7]. It abstracts the computational processes of chemical reactions that occur both within membrane-bound compartments and across membranes. The resulting computational framework is known as the P system, which utilizes hierarchical membrane structures to perform computations in a massively parallel and distributed manner. Notably, P systems have been proven to possess computational capabilities equivalent to those of Turing machines, enabling them to efficiently solve complex problems. Leveraging the exceptional performance and inherent parallelism of P systems, researchers have developed membrane-inspired evolutionary algorithms (MIEAs). These algorithms integrate membrane structures, evolutionary rules, and computational mechanisms from membrane computing with the search principles of meta-heuristic algorithms. The evolutionary processes within the membrane architecture are referred to as algorithms in membranes. MIEAs exploit the parallel processing capa-

bilities of P systems to enhance search efficiency and solution quality. Over the past two decades, numerous MIEAs have been proposed and successfully applied to a wide range of complex real-world optimization problems. This body of work demonstrates the effectiveness and versatility of MIEAs in addressing challenging computational tasks while highlighting their potential as powerful tools in evolutionary computation research [8].

Given the exceptional computational capabilities and inherent parallelism of membrane computing systems, it is both logical and promising to extend MIEAs to address multi-objective multi-task optimization (MOMTO) problems. Although MIEAs have demonstrated considerable effectiveness in solving complex optimization tasks, their potential application in the realm of multi-task optimization has been little explored. To address this gap, we propose a novel evolutionary framework that integrates membrane computing principles with multi-task optimization strategies specifically designed for MOMTO problems. This framework leverages the hierarchical membrane structures and parallel processing capabilities of P systems to handle multiple optimization tasks concurrently. By combining the strengths of membrane computing with EMT, the proposed approach aims to enhance the efficiency and generalization performance of evolutionary algorithms when solving MOMTO problems, thereby contributing a significant advancement to the field. In summary, the main contributions of this paper are as follows:

- (1) A novel membrane-inspired evolutionary algorithm with a hybrid membrane structure is proposed for solving MOMTO problems.
- (2) A transfer information source construction method based on knee point solutions is proposed, enhancing the efficiency of information transfer between different tasks and avoiding negative transfer.

The remainder of the paper is organized as follows. [Section 2](#) reviews related work. [Section 3](#) describes the proposed algorithm. [Section 4](#) presents experimental results validating the effectiveness of the algorithm. [Section 5](#) concludes the paper and outlines future research directions.

## 2 Related Work

### 2.1 Multi-objective multi-task optimization

Multi-task optimization involves the simultaneous optimization of multiple tasks by leveraging shared knowledge among them to enhance overall performance. Formally, given  $K$  optimization tasks formulated as minimization problems, the objective is to find the optimal solutions  $\{\mathbf{x}_1^*, \mathbf{x}_2^*, \dots, \mathbf{x}_K^*\}$  such that:

$$\mathbf{x}_j^* = \arg \min_{\mathbf{x}_j} T_j(\mathbf{x}_j), j = 1, 2, \dots, K \quad (2)$$

where  $T_j(\mathbf{x}_j)$  represents the objective function(s) of task  $T_j$ . When at least one task is a multi-objective optimization problem (MOP), the overall problem is referred to as a multi-objective multi-task optimization problem. EMT algorithms have been developed to address MTO by facilitating knowledge transfer between tasks. A notable example

is the Multifactorial evolutionary algorithm (MFEA) [5], which treats each task as an independent environmental influence and introduces the concept of a skill factor to denote the task in which an individual exhibits the best performance. The key mechanisms in MFEA include vertical cultural transmission, where offspring inherit the skill factors of their parents, and assortative mating, which controls the intensity of cross-task information transfer through a parameter known as the random mating probability. While these strategies enable effective information sharing, they also present challenges such as negative transfer and inefficient resource allocation. To overcome these limitations, we propose a method based on knee point solutions to construct the transfer information source, enhancing the efficiency of information transfer between tasks and mitigating negative transfer.

## 2.2 Membrane-inspired evolutionary algorithms

MIEAs are a novel class of heuristic algorithms that integrate the principles of P systems with evolutionary rules and computational mechanisms. These algorithms draw inspiration from distinct P system architectures and are broadly categorized into hierarchical structure MIEAs, modeled on cell-like P systems, and networked structure MIEAs, influenced by tissue-like P systems. The fundamental components of MIEAs encompass membrane structures, reaction rules, and multisets, which collectively emulate biological processes for optimization tasks. Membrane structures define hierarchical computational regions analogous to cell membranes while reaction rules drive the evolution and interaction of symbolic objects within these regions. These mechanisms empower MIEAs to effectively address diverse and intricate optimization challenges [8].

Different hierarchical configurations of MIEAs include nested membrane structures (NMS) [9], one-level membrane structures (OLMS) [10], hybrid membrane structures (HMS) [11], and dynamic membrane structures (DMS) [12]. NMS organize membranes in nested layers where superior solutions propagate inward to deeper layers while inferior solutions move outward, facilitating stepwise refinement of results. In contrast, OLMS distribute multiple elementary membranes within a skin membrane with each membrane independently evolving solutions. Communication rules allow the exchange of the best solutions between these elementary membranes and the skin membrane, achieving a balance between exploration and exploitation. HMS combines the features of NMS and OLMS by embedding multiple NMS structures within a single OLMS framework, enhancing adaptability and computational diversity. On the other hand, DMS employs dynamic structural changes through division and dissolution rules that enable system adjustment during evolution to adopt different strategies at various stages effectively balancing exploration and exploitation. The algorithm proposed in this paper is based on HMS.

## 3 Proposed algorithm

In this section, the proposed Knee-MOMTMIEA is introduced.

**Algorithm 1:** The overall framework of Knee-MOMTMIEA

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**Input:**  $K$ : the number of tasks.  
**Output:** the final multisets.

- 1 **Initialization**
- 2 **while** the stop criterion is not met **do**
- 3     **for**  $k$ th inner-membrane in  $K$  inner-membrane **do**
- 4         **for** each symbol object  $x$  in the  $k$ th inner-multiset **do**
- 5             **Inter-task Evolution**
- 6             **Knee solution-based information transfer**
- 7             **Selection**
- 8 Call **Dissolution rule** to release all the inner-membranes to the skin membrane.

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**3.1 The framework of Knee-MOMTMIEA**

The main framework of the Knee-MOMTMIEA is summarized in [Algorithm 1](#), and its flowchart is illustrated in [Fig. 1](#). First, the skin membrane and  $K$  sub-membranes are initialized with their respective initial multisets. Then, for each sub-membrane, the evolutionary process is continuously conducted. Specifically, the evolutionary process consists of three main steps. The first step involves performing *inter-task evolution* for each symbol object. In the second step, after one complete iteration, the *knee solution-based information transfer* is applied to exchange objects between different task membranes to achieve effective information transfer. The third step selects high-quality symbol objects to proceed to the next generation. Finally, when the termination condition of the algorithm is met, all symbol objects are output to the skin membrane.

**3.2 Initialization**

The core of the initialization process lies in constructing a membrane structure system tailored for multi-task optimization and allocating corresponding computational resources to each task to support the subsequent evolution of symbol objects. First, the membrane structure is initialized by invoking the **division rule** to divide the skin membrane into  $K$  inter-membranes. Here,  $K$  represents the number of tasks in the multi-task optimization problem, and each inter-membrane is dedicated to optimizing a specific task. The **division rule** is defined as follows:

Division rule :

$$\llbracket \_ \rrbracket_0 \longrightarrow [\llbracket \_ \rrbracket_1, \llbracket \_ \rrbracket_2, \llbracket \_ \rrbracket_3, \dots, \llbracket \_ \rrbracket_k, \dots, \llbracket \_ \rrbracket_{K-1}, \llbracket \_ \rrbracket_K]_0 \quad (1)$$

Next, all symbol objects are evenly distributed across the  $K$  sub-membranes. Each sub-membrane  $\llbracket \_ \rrbracket_k$  receives  $\lceil \frac{N}{K} \rceil$  symbol objects, and a skill factor  $k$  is assigned to each object corresponding to the  $k$ -th task. Subsequently, the symbol objects within each sub-membrane are randomly initialized. For the  $k$ -th task, each symbol object in the sub-membrane is randomly initialized within the upper and lower bounds of that task. Finally, each symbol object is evaluated based on its corresponding skill factor.

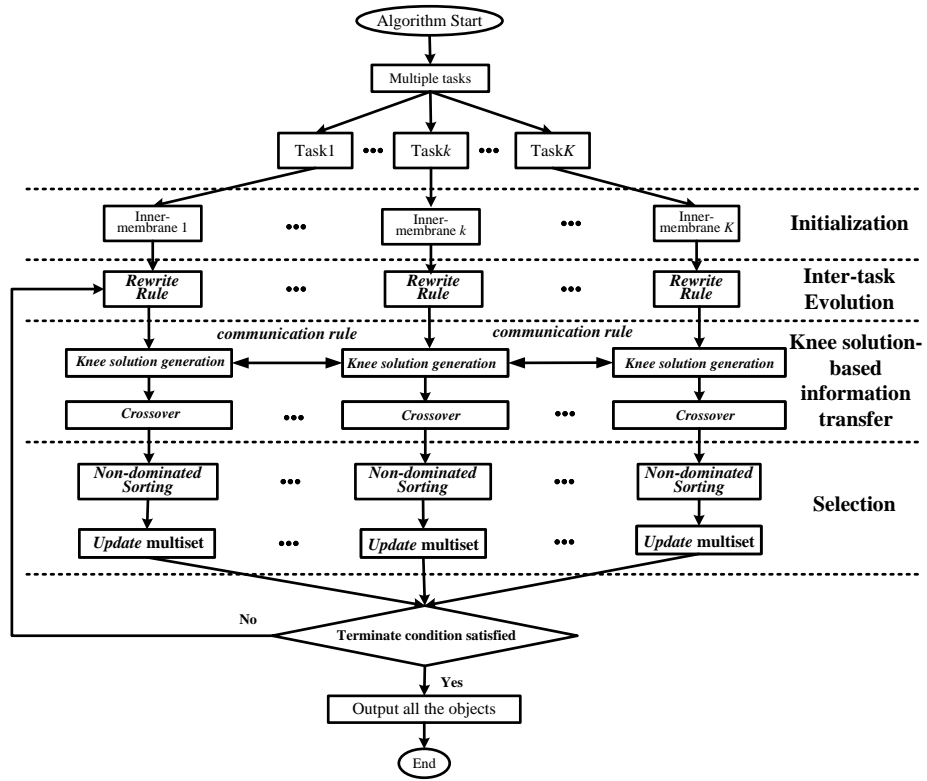


Fig. 1. The flowchart of the proposed Knee-MOMTMIEA.

### 3.3 Inter-task Evolution

Inter-task evolution is dedicated to independently evolving the multiset within each inner membrane. Each symbol object undergoes reproduction by randomly applying one of three rewriting rules to generate offspring. These three rewrite rules are adaptations of the differential evolution operator, designed to exhibit different search behaviors, as detailed below.

Rewrite rule 1:

$$\begin{aligned} \mathbf{v} &= \mathbf{x}_{\text{best}} + F_1 \cdot (\mathbf{x}_{\text{rand1}} - \mathbf{x}_{\text{rand2}}) \\ \mathbf{o} &= \text{crossover}(\mathbf{v}, \mathbf{x}) \end{aligned}$$

Rewrite rule 2:

$$\begin{aligned} \mathbf{v} &= \mathbf{x}_{\text{rand1}} + F_1 \cdot (\mathbf{x}_{\text{rand2}} - \mathbf{x}_{\text{rand3}}) \\ \mathbf{o} &= \text{crossover}(\mathbf{v}, \mathbf{x}) \end{aligned} \quad (2)$$

Rewrite rule 3:

$$\begin{aligned} \mathbf{v} &= \mathbf{x} + F_2 \cdot (\mathbf{x}_{\text{rand1}} - \mathbf{x}) + F_1 \cdot (\mathbf{x}_{\text{rand2}} - \mathbf{x}_{\text{rand3}}) \\ \mathbf{o} &= \text{crossover}(\mathbf{v}, \mathbf{x}) \end{aligned}$$

Where  $\mathbf{x}$  represents the parent symbol object,  $\mathbf{o}$  denotes the offspring symbol object, and  $\mathbf{v}$  refers to the trial vector. The SBX [13] operator is employed for crossover operations. Additionally,  $rand1, rand2$ , and  $rand3$  are random numbers indicating the indices of randomly selected symbol objects for the crossover operation.  $F_1$  and  $F_2$  are scaling factors within the range  $(0, 1)$ , while  $\mathbf{x}_{best}$  is a randomly chosen non-dominated solution from the current generation's multiset of non-dominated solutions. The first rewrite rule possesses a strong global search capability, enabling the algorithm to achieve a higher convergence rate. However, its performance in local search is relatively weaker. The second rewrite rule, while having a slower global search speed, demonstrates superior local search ability. The third rewrite rule adopts a rotation-invariant strategy, which is specifically designed to address multi-objective problems effectively. During the early stages of the evolutionary selection process, the first rewrite rule is particularly advantageous due to its high efficiency and rapid search capability, making it suitable for exploring a wide solution space. In contrast, during the later stages of evolution, the second rewrite rule becomes more effective as it promotes diversity in the population and facilitates the generation of a larger set of non-dominated solutions, thus enhancing the quality of the final solutions.

### 3.4 Knee solution-based information transfer

This strategy focuses on knowledge transfer and sharing across tasks. Specifically, the knee solution from each inner membrane is first selected as a representative individual of the entire task multiset to participate in knowledge sharing. Then, the communication rule is applied to transmit the knee solutions to other tasks. Finally, information transfer is achieved by performing crossover operations between the knee solution and symbol objects from other tasks.

Knee solutions represent a special class of solutions in multi-objective optimization, located at the knee points of the Pareto front. In these regions, a small improvement in one objective often results in a significant deterioration in another objective. Thus, knee solutions represent the best trade-offs between conflicting objectives. They provide decision-makers with a focal point, enabling the filtering of solutions that have relatively less impact on decision-making from the large Pareto front, thereby reducing the solution space. Consequently, selecting knee solutions as representatives of task multisets is both efficient and reasonable. In the proposed algorithm, the knee solution is determined based on the normalized non-dominated solution set in the objective space by identifying the solution with the minimum Manhattan distance. The Manhattan distance is computed using the following formula:

$$MD(\mathbf{x}) = \sum_{i=1}^M \left| \frac{f_i(\mathbf{x}) - f_i^{min}}{f_i^{max} - f_i^{min}} \right| \quad (3)$$

In this context,  $M$  represents the number of objective functions.  $f_i^{min}$  and  $f_i^{max}$  denote the minimum and maximum values of the  $i$ -th objective after normalization, respectively. The solution with the smallest Manhattan distance is identified as the knee solution.

### 3.5 Selection

The selection process is based on a non-dominated sorting approach. Specifically, the parent multiset is combined with the individuals generated through *inter-task evolution* and *knee solution-based information transfer* to form a new multiset. The advantage solutions for each task are then retained by applying non-dominated sorting and using the crowding distance metric to maintain diversity and select high-quality solutions.

## 4 Experiment

In our experimental analysis, the proposed EMT-MOMIEA is comprehensively evaluated to assess its effectiveness in solving multi-objective multi-task problems. The algorithm’s performance is systematically compared against three state-of-the-art multi-objective EMT algorithms—MOMFEA-II [14], MOMFEA [15], and MFEA-GHS [16], as well as the widely recognized single-task multi-objective optimization algorithm, NSGA-II [17]. The experiments are conducted on the classical MOMTO test suites [18].

### 4.1 Test suites introduction

The efficiency of information transfer between tasks is primarily influenced by two critical factors: the degree of overlap in the global optima and the similarity of the tasks’ fitness landscapes. When there is a close alignment between optimal solutions of both source and transfer tasks, shared evolutionary progress can benefit all involved tasks. Similarly, if the fitness landscapes exhibit high similarity across tasks, transferred information is more likely to accelerate convergence. To systematically evaluate these characteristics, problems in the classical MOMTO test suite [18] are categorized based on two dimensions: intersection degree of global optima and similarity of fitness landscapes. The intersection degree categorizes optimal solution overlap into three categories: complete intersection (CI), partial intersection (PI), and no intersection (NI). Additionally, fitness landscape similarity is classified as high similarity (HS), medium similarity (MS), or low similarity (LS) to reflect alignment levels in search spaces across tasks. By combining these dimensions, the classical MOMTO test suite comprises nine sub-problems that span various task interactions labeled from CIHS to NILS. These sub-problems provide a comprehensive benchmark for evaluating EMT algorithm performance in diverse multitasking scenarios while ensuring robust assessment capabilities for handling varying levels of overlap and similarity between tasks.

### 4.2 Compared algorithms and parameter settings

MOMFEA [15], as the first EMT algorithm for multi-objective optimization, integrates NSGA-II [17] as its evolutionary operator within the multitasking framework, establishing a foundational benchmark. MOMFEA-II [14] enhances this framework with an adaptive online learning mechanism to dynamically adjust information transfer intensity, improving task-specific performance. MFEA-GHS [16] introduces genetic transformation and hyper-rectangle search to improve solution transfer across tasks

and optimize search within constrained regions. NSGA-II, a widely recognized single-task algorithm, serves as a baseline for evaluating the advantages of multitasking approaches.

All algorithms are implemented using the JMetal 4.5.2 [20] framework to ensure consistency and eliminate platform-based discrepancies. The experiments are conducted on a machine with an Intel Core i5-9400F CPU (2.90GHz) and 16GB of RAM. The population size is set to 100 for NSGA-II in the classical MOMTO test suites, while it is set to 200 for all EMT algorithms.

For reliability, each algorithm is independently executed 20 times on all test suites, with results summarized as the mean Inverted Generational Distance (IGD) [19] values. In result tables, the best performance on each sub-problem is highlighted in gray. Statistical significance is analyzed using the Wilcoxon rank-sum test at a 95% confidence level, with symbols "+", "-", and "=" indicating better, worse, or equivalent performance, respectively.

### 4.3 Experimental results and analysis

The comparative results on the classical MOMTO test suite, as presented in Table 1, highlight the superior performance of the proposed Knee-MOMTMIEA against state-of-the-art algorithms. Multi-objective EMT algorithms outperform single-task NSGA-II on most test problems, demonstrating the effectiveness of knowledge transfer mechanisms in improving optimization efficiency. Specifically, Knee-MOMTMIEA achieves superior results on 14, 15, 16 and 16 sub-problems compared to MOMFEA-II, MOMFEA, MFEA-GHS and NSGA-II respectively out of a total of 18 sub-problems. These findings validate the robustness and efficiency of our method across diverse multitasking scenarios. For problems with high similarity such as CIHS-1, PIHS-1 and NIHS-1 tasks; Knee-MOMTMIEA consistently outperforms its competitors due to an adaptive information transfer strategy based on knee solutions that effectively facilitates knowledge exchange between tasks with closely aligned fitness landscapes. By leveraging knee solutions as representatives for information transfer; our algorithm accelerates convergence and improves solution quality particularly when dealing with highly correlated fitness landscapes. Furthermore; our algorithm demonstrates remarkable effectiveness on no-intersection problems such as NIHS-2; NIMS-1 and NILS-2 where global optima do not overlap among tasks resulting in minimal task similarity. These types of problems are inherently more challenging due to lack of shared information between tasks but Knee-MOMTMIEA adaptively controls useful knowledge transfer using a knee solution-based mechanism ensuring effective information exchange even when tasks are highly dissimilar enabling robust optimization across a wide range of scenarios.

## 5 Conclusion

In this paper, we proposed the Knee solution-based membrane-inspired evolutionary algorithm to effectively address multi-objective multi-task optimization problems. By combining membrane computing principles with evolutionary multitasking strategies,

**Table 1.** Mean values of the IGD obtained by five algorithms on the classical MOMTO test problems.

Problem	MOMFEA-II	MOMFEA	MFEA-GHS	NSGA-II	Knee-MOMTMIEA
CIHS-1	4.48E-04(+)	4.83E-04(+)	2.36E-03(+)	2.24E-03(+)	3.92E-04
CIHS-2	2.56E-03(+)	2.37E-03(+)	4.63E-03(+)	4.43E-03(+)	3.90E-04
CIMS-1	1.86E-01(+)	4.84E-02(+)	1.83E-01(+)	1.11E-01(+)	4.20E-03
CIMS-2	3.93E-04(-)	5.99E-03(+)	6.07E-02(+)	3.20E-02(+)	5.22E-03
CILS-1	3.83E-04(+)	4.03E-04(+)	3.10E-03(+)	2.91E-01(+)	3.53E-04
CILS-2	4.67E-04(+)	3.75E-04(+)	3.22E-03(+)	2.15E-04(+)	3.41E-04
PIHS-1	4.21E-04(+)	5.54E-04(+)	2.67E-03(+)	1.50E-03(+)	4.14E-04
PIHS-2	1.53E-02(+)	1.42E-02(+)	4.18E-01(+)	5.02E-02(+)	9.01E-03
PIMS-1	2.27E-03(-)	3.51E-03(-)	9.72E-02(+)	6.43E-03(-)	2.54E-02
PIMS-2	1.56E+01(+)	1.12E+01(+)	2.20E-03(-)	1.06E+01(+)	4.85E+00
PILS-1	4.47E-04(+)	5.57E-04(+)	2.47E-03(+)	3.14E-04(+)	3.15E-04
PILS-2	5.56E-03(-)	7.54E-03(-)	6.85E-01(+)	6.08E-01(+)	4.54E-01
NIHS-1	1.73E+00(+)	1.69E+00(+)	1.53E+01(+)	6.48E+00(+)	1.54E+00
NIHS-2	4.45E-04(+)	5.55E+04(+)	2.37E-03(+)	7.59E-04(+)	3.56E-04
NIMS-1	3.96E-01(+)	3.33E-01(+)	2.87E+00(+)	5.53E-01(+)	2.38E-01
NIMS-2	1.54E-02(-)	3.74E-02(+)	3.89E-03(-)	1.69E-01(+)	3.09E-02
NILS-1	1.20E-03(+)	1.06E-03(-)	4.65E-02(+)	8.45E-04(-)	1.12E-03
NILS-2	6.67E-01(+)	6.39E-01(+)	1.66E-02(+)	6.93E-01(+)	1.02E-03
	+14/-4/=0	+15/-3/=0	+16/-2/=0	+16/-2/=0	

the algorithm utilizes hierarchical membrane structures and knee solution-based information transfer to enhance optimization performance. The knee solution mechanism ensures adaptive and efficient knowledge transfer across tasks, improving convergence and solution diversity while minimizing the risks of negative transfer. The proposed algorithm was evaluated on the classical MOMTO test suite and consistently outperformed state-of-the-art EMT algorithms and the single-task NSGA-II. These results validate the robustness and efficiency of Knee-MOMTMIEA in handling diverse multitasking scenarios, including tasks with high similarity and no intersection. This work provides an effective and innovative framework for solving MOMTO problems, advancing the integration of membrane computing and evolutionary multitasking techniques.

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