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Cultural transmission based multi-objective evolution strategy for evolutionary multitasking

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ABSTRACT

In recent years, many efficient evolutionary multitasking (EMT) algorithms have been proposed to solve multi-objective multi-task optimization problems. However, EMT algorithms often face negative transfer problems. In this paper, a novel multi-objective evolution strategy, called CT-EMT-MOES, is proposed based on a cultural transmission theory for solving multi-objective multitask optimization problems. First, two evolutionary operators inspired by cultural transmission theory are proposed. The elite-guided variation strategy can transfer the information from the current Pareto front to all individuals and guide the population to quickly converge. The horizontal cultural transmission strategy can efficiently transfer information from the source task to the target task. Second, to solve the negative transfer problem, an adaptive information transfer strategy is proposed to adaptively adjust the probability of an information transfer. Third, the proposed algorithm can gain a Pareto front with good convergence and diversity by utilizing a smaller population size and fewer computing resources. As a result, the proposed algorithm can effectively utilize the implicit similarity and complementarity between simultaneous optimized tasks to improve the overall convergence efficiency and reduce a negative transfer. Finally, comprehensive experimental results show that the proposed algorithm can achieve a better performance compared with previous state-of-the-art multi-objective EMT algorithms.

1. Introduction

Multi-objective optimization problems (MOPs) are widespread in scientific and engineering applications [1], aiming to optimize multiple conflicting objective functions simultaneously. For the sake of generality, assuming that the problem is a minimization problem, MOP can be expressed as Eq. (1).

$$\underset{x\in\Omega}{\mathrm{Min}}\, \mathrm{F}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x}), \cdots, f_M(\mathbf{x}))^T \tag{1}$$

where $\mathbf{x} = (x_1, x_2, x_3, \cdots, x_D)\epsilon\Omega$ expresses a D-dimension decision vector within the feasible region of the decision space Ω. The mapping function $\mathrm{F}(\mathbf{x}):\Omega \rightarrow R^M$ defines the objective function, and R^M indicates the objective function space. Because the objective functions conflict with each other, it is impossible to seek a single optimal solution that satisfies all objective

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functions. Therefore, the definition of Pareto optimality is applied to obtain the best trade-off solutions. Given two solutions, **x** and **y**, if **x** is not inferior to **y** for all objectives and performs better than **y** on at least one objective, it is **x** called Pareto dominates **y**, which is expressed as $\mathbf{x} \prec \mathbf{y}$. If **x** has advantages over **y** on certain objectives, but **y** is also better than **x** on at least one objective, it is said that **x** and **y** are Pareto non-dominated. If the solution $\mathbf{x}^*$ is not Pareto dominated by any other solution in the current population, then $\mathbf{x}^*$ is called a Pareto optimal solution. The set of all Pareto optimal solutions is the Pareto optimal solution set (PS). The corresponding projection of PS in the objective function space is known as the Pareto front (PF).

Evolutionary computations have become the mainstream method for solving MOPs because of their powerful search performance of evolutionary algorithms [47–49]. Conventional multi-objective evolutionary algorithms (MOEAs) can be mainly divided into three categories: Pareto dominance relationship based algorithms [2–4], decomposition of objective function based algorithms [5–7], and evaluation indicator based algorithms [8–10]. After a limited number of iterations, MOEAs can search for an approximate PS. However, many real-world MOPs are related, and the knowledge gained from optimizing a problem should be effectively utilized to solve similar problems.

In recent years, inspired by the ability of the human brain, evolutionary algorithms have been applied to process multiple tasks simultaneously and reuse the knowledge learned from different tasks. Gupta et al. [11] proposed the evolutionary multitasking (EMT) concept. A universal algorithm framework namely multifactorial algorithm for solving multi-task optimization (MTO) problems was also proposed, and an instance of embedding the genetic algorithm into the multifactorial framework (MFEA) was implemented. The original intention of the EMT is to adopt the advantages of the implicit similarity and complementarity of multiple simultaneously optimized tasks to efficiently reuse the knowledge obtained in other tasks and improve the overall optimization efficiency. In recent years, numerous EMT algorithms have been proposed for solving MTO problems, which can be broadly classified into five different categories: improving the search strategy approaches [16–19], optimal source task selection approaches [20–23], improving knowledge transfer strategy approaches [24–27], allocating search resource approaches [28–30], and search space mapping approaches [31–34].

However, existing EMT algorithms often face several challenges. First, most ignore the vast differences in simultaneously processing tasks that are not always highly similar. When two unrelated problems are simultaneously optimized, it often leads to a negative transfer. That is, the optimization effect of the target task worsens after an information transfer. Second, they generally do not consider the timing of the information transfer. An improper transfer timing degrades the performance. For example, transferring any other information into a population that is already very close to the true PF will result in a negative transfer to the target task. Third, most studies have not considered the probability of an information transfer. A large amount of information exchange between similar tasks can make both quickly converge. By contrast, the exchange of information between completely different tasks should be moderate. Finally, most need to maintain a large population size because every sub-problem needs a sufficient sub-population to find and maintain the optimal solutions.

To address these issues, a novel multi-objective evolutionary multitasking evolution strategy based on cultural transmission was proposed, namely, CT-EMT-MOES. The proposed algorithm falls into the category of improving knowledge transfer strategy approaches. Inspired by the modern cultural evolution theory [39], each individual can be updated by two evolutionary engines named elite-guided variation strategy and horizontal cultural transmission strategy. The elite-guided variation strategy can transfer the current PF information to all individuals and guide the population to quickly converge. Moreover, the horizontal cultural transmission strategy operates under the assumption that, to progress, the current population should also appropriately accept external cultural influences. The horizontal cultural transmission strategy can bring richer diversity to the current population and appropriately promote the convergence of the target task population. To make full use of the two transfer strategies, an adaptive information transfer strategy is proposed to adaptively adjust the probability of the information transfer according to the dominant relationship between the offspring and its parent and thus reasonably allocate the evolutionary resources. In addition, the evolutionary strategy search engine suffices to guide the population based only on the dominant relationship between the offspring and its parent and can cover the true PF even with a small population size. As the population size increases, the PF obtained becomes more specific. This characteristic makes the evolution strategy extremely efficient when solving sub-problems that can only be allocated few search resources from the total population. Comprehensive experiments were carried out on the classical and complex multi-objective multi-task optimization (MOMTO) benchmark test suites to verify the effectiveness of the proposed CT-EMT-MOES. The experimental results demonstrate that the proposed CT-EMT-MOES is superior to or comparable with other advanced EMT algorithms. In summary, the contributions of this study are summarized as follows.

(1) An evolution strategy based on a cultural transmission is employed to deal with multi-objective multi-task problems. Inspired by cultural transmission theory, two operators, an elite-guided variation strategy and a horizontal cultural transmission strategy, are proposed to realize the information transfer within and between tasks.
(2) An adaptive information transfer strategy is proposed to adaptively adjust the probability of an information transfer between tasks and reasonably allocate the evolution resources of information transfer within and between tasks to increase the positive transfer and thereby avoid negative transfer.
(3) To verify the effectiveness of the proposed CT-EMT-MOES, comprehensive experiments were carried out on the classical and complex multi-objective multi-task optimization benchmark test suites. The experimental results demonstrate that the proposed CT-EMT-MOES is superior to or comparable to other advanced EMT algorithms. A

comparative experiment was conducted on the proposed operators to prove their effectiveness. In addition, the proposed CT-EMT-MOES achieved remarkable results with a small population.

The remainder of this paper is organized as follows. Section 2 provides a comprehensive analysis of related studies. Section 3 describes the details of the proposed CT-EMT-MOES method. Section 4 presents comprehensive experiments to assess the effectiveness of the CT-EMT-MOES. Finally, Section 5 provides some concluding remarks and discusses areas of future direction.

2. Related work and motivation

2.1. Multi-task optimization

Conventional optimization problems can be divided into single-objective optimization problems (SOPs) and MOPs according to the number of objective functions. SOPs are dedicated to finding the optimal solution, and MOPs are devoted to searching for a set of tradeoff solutions. Nevertheless, they are both single-task problems.

The multi-task optimization proposed by Gupta et al. [11] is committed to solving multiple optimization problems simultaneously using the potential complementarity and similarity between problems. For the sake of generality, assuming that K minimization problems are optimized simultaneously, the formal representation of MTO problems is as Eq (2).

$$\{\mathbf{x}_1^*, \mathbf{x}_2^*, \cdots, \mathbf{x}_K^*\} = \{\mathrm{argmin}T_1(\mathbf{x}_1), \mathrm{argmin}T_2(\mathbf{x}_2), \cdots, \mathrm{argmin}T_K(\mathbf{x}_K)\} \tag{2}$$

where $T_j(j = 1, 2, \ldots K)$ denotes the jth task to be optimized, $\mathbf{x}_j$ represents the feasible solution of the jth task, and $\mathbf{x}_j^*$ indicates the optimal solution or solution set of the jth task. When there is at least one MOP among multiple simultaneous optimization tasks, this problem is called a multi-objective multitask optimization problem.

It should be noted that although MOPs and MTO problems both optimize multiple objective functions simultaneously, they are two completely different optimization problem paradigms. MOPs only solve one task at a time, and there is only one decision space. By contrast, MTO problems require a simultaneous optimization of multiple completely different tasks. Therefore, there are multiple different decision spaces in the MTO, and each decision space corresponds to an independent task. MTO problems have greatly expanded the field of optimization problems. When the MTO problem contains only one optimization sub-problem, it can be considered a general SOP or MOP. Traditional SOPs and MOPs can be considered a subset of MTO problems.

2.2. Evolutionary multitasking

To solve MTO problems, inspired by the multitasking learning [12] and transfer learning [13] theory in the machine learning area and the multifactorial inheritance concept [14,15] in culturology. Gupta et al. [11] proposed the first universal EMT algorithm framework, namely, MFEA, where each task is considered a memetic factor influencing the individual evolution in a K-factorial environment. To achieve information transfer between the decision spaces, MFEA proposed the unified decision space theory that the decision vectors of different tasks simultaneously optimized will be encoded into a unified decision space with the same upper and lower bounds and the number of dimensions. Following Darwinism, MFEA put forward the vertical cultural transmission theory indicating that offspring will inherit the tasks of their parents. To control the intensity of information transfer, MFEA proposed the assortative mating theory in which individuals who are superior at the same task can freely mate. Otherwise, hybridization is necessary to meet the artificially set probability random mating probability (rmp). To evaluate the performance of an individual when dealing with different tasks, a series of indicators is defined in the MFEA. For example, the factorial rank r_j^i is defined as the ranking of the ability of $\mathbf{x}_i$ to solve the sub-problem T_j within the entire population. Scalar fitness φ_i represents the best performance that individual $\mathbf{x}_i$ can achieve in all tasks, calculated according to the factorial rank of $\mathbf{x}_i$, i.e., $\varphi_i = \frac{1}{min_{j\in\{1,2\cdots k\}} r_j^i}$. The skill factor τ_i indicates the most suitable task for individual $\mathbf{x}_i$, that is, the task index for $\mathbf{x}_i$ to obtain the best factorial rank, which is denoted as $\tau_i = argmin_j\left\{r_j^i\right\}$. In general, MFEA provides a basic framework for solving MTO problems. These concepts have a profound impact on the subsequent EMT algorithms. The current EMT algorithms can be classified into five categories: approaches improving the search strategy [16–19], optimal source task selection approaches [20–23], approaches improving the knowledge transfer strategy [24–27], approaches allocating search resources [28–30], and search space mapping approaches [31–34].

2.2.1. Improving search strategies approaches

The introduction of distinctive search operators into the EMT framework can provide the ability to solve a specific problem. A variety of search operators can bring different evolutionary possibilities to the population for significantly exploring and exerting the superiority of the EMT. Gupta et al. [16] applied for the first time the popular fast and elitist multi-objective genetic algorithm (NSGA-II) [2] as an evolutionary engine into the MFEA framework to extend the EMT algorithm to the multi-objective optimization field and proposed a multi-objective multifactorial algorithm (MOMFEA). Liang et al. [17] proposed a novel multifactorial algorithm, called MFEA-GHS, based on a hyper-rectangle search strategy and a genetic transform strategy. The hyper-rectangle search strategy based on opposition learning is dedicated to improving the search ability in each subspace, and the genetic transform strategy can construct a mapping vector to map variables with different boundaries to a similar space. In addition, Yu et al. [18] proposed an EMT algorithm-integrated opposition-based learning and differential evolution (DE), utilizing the complementarity of the search neighborhoods of these two operators to increase the search scope. Liu et al. [19] also presented a surrogate-assisted multitasking memetic algorithm, where each task employs the DE as the global optimization operator, the covariance matrix adaptation evolution strategy as the local search operator, and the surrogate model with the Gaussian process to predict the optimal solution and enhance the search efficiency.

2.2.2. Optimal source task selection approaches

This type of algorithm is devoted to selecting the most appropriate task as the source task to improve the information transfer efficiency when there are more than two tasks to be optimized simultaneously. In addition, Chen et al. [20] proposed a novel adaptive archive-based EMT algorithm for many-task optimization named MaTDE. MaTDE introduced an archive strategy to measure the similarity between tasks by calculating the Kullback–Leibler divergence of the solution distribution. By combining the accumulated reward mechanism, the algorithm can select the optimum source task. Huang et al. [21] also proposed an adaptive knowledge transfer method, in which the covariance matrix was utilized to characterize the distribution of the historical solutions of a specific task. The similarity of the covariance matrix is the basis for selecting the most suitable task, and a pheromone approach is applied to adjust the selection probability dynamically. Shang et al. [22] suggested an adaptive task selection mechanism based on a credit assignment, in which the source tasks that provide more beneficial knowledge to the target task will obtain more transfer opportunities during the evolution process. Zhang et al. [23] presented a source-task selection framework with three different selection strategies. The probabilistic model learned using the estimation distribution algorithm represents the distribution of solutions in a specific task, and the Wasserstein distance is applied to evaluate the similarity between tasks.

2.2.3. Improving knowledge transfer strategy approaches

This type of algorithm is dedicated to dynamically changing the transferred information and the frequency of information transfer based on the performance of the algorithm on different tasks during a simultaneous optimization. Li et al. [24] proposed an adaptive transfer intensity strategy. The intensity of knowledge transfer is positively related to the transfer success rate, which represents the proportion of offspring generated by transferring methods that are superior to their parents. Zhou et al. [25] studied the influence of different types of crossover operators on knowledge transfer. A novel MFEA algorithm with adaptive knowledge transfer (MFEA-AKT) was proposed to improve the robustness of search performance in different problems. The crossover operators in MFEA-AKT can be adaptively adjusted based on the knowledge transfer information collected in the evolution process. Bali et al. [43] further analyzed the theoretical basis of MFEA and pointed out that the degree of knowledge transfer is adjusted based on a mixture of probabilistic models. And a fixed parameter *rmp* in conventional MFEA is the cause of the negative transfer phenomenon. To obtain the optimal transmission matrix, they proposed a novel EMT algorithm based on online transfer parameter estimation named MFEA-II. Subsequently, they applied this online learning method to adjust the transfer strength matrix to multi-objective optimization and proposed a novel multi-objective EMT algorithm called MOMFEA-II [26]. Wang et al. [27] deemed that in a complex multi-task environment, the potential relationships between the target task and the highly diversified individuals in the source task directly determine the effectiveness of knowledge transfer. They introduced an anomaly detection model into the MTO field to detect individuals who have common knowledge with the target task and proposed a novel EMT algorithm based on anomaly detection (MTEA-AD). Specifically, each task is assigned a sub-population and an anomaly detection model. Each anomaly detection model is used to learn the relationship between individuals in the target task and other tasks and detect individuals who may carry negative knowledge and candidate transfer individuals who can assist the target task.

2.2.4. Allocating search resources approaches

Simultaneous tasks generally have different degrees of difficulty. Based on the idea that more search resources should be allocated to more difficult tasks, EMT algorithms based on the allocation of the search resources have been proposed. Wen et al. [28] proposed a mechanism for reallocating computational resources to tasks when the parting-way phenomenon, defined as a knowledge transfer, begins to fail. In addition, Yao et al. [29] presented an EMT algorithm based on a decomposition and dynamic resource allocation strategy. The subproblem with a faster evolution rate allocates more computational resources. Gong et al. [30] proposed an online dynamic resource allocation strategy based on the problem difficulty, such that more complex problems will be allocated more resources.

2.2.5. Search space mapping approaches

These EMT algorithms are proposed to solve the problem of simultaneous optimization tasks having heterogeneous decision spaces, which may lead to a negative transfer. Ding et al. [31] introduced a novel decision variable translation strategy into an EMT algorithm that can map the solutions in different problem spaces into a unified space to reduce the differences in location. Bali et al. [32] presented a linearized domain adaptation method to transform the search space of a simple task to a reconstruction space that is highly correlated with a complex task to reduce the discrepancy of search spaces among tasks to provide a platform for an efficient knowledge transfer. Feng et al. [33] proposed a novel EMT algorithm based on a denoising autoencoder task mapping mechanism (EMT-A). The solutions to the source task can be projected onto the target task by the mapping matrix learned by the denoising autoencoder. This explicit transfer method can avoid constraining the decision spaces of different tasks from a unified representation, allowing them to be more flexible and diverse. Liang et al. [34] introduced the subspace alignment strategy and adaptive differential evolutionary operator in a novel multi-objective multifactorial algorithm, namely, MFEA-SADE. The subspace alignment strategy can unify the search space of different tasks by generating a mapping matrix, thereby reducing the negative transfer. The improved adaptive differential evolution can provide numerous search strategies to improve the search efficiency.

2.3. Motivation

EMT algorithms aim to transfer helpful information from the source task to the target task to help it converge, and this type of transfer is called a positive information transfer. However, EMT algorithms often face negative transfer problems. After the information transfer of the simultaneously optimized tasks, the optimization effect is worse than optimization separately.

In the transfer learning area [12,13], the parameters of the source model are always kept to reuse the learned knowledge. Inspired by this strategy, the population optimized by well-known MOEA like NSGA-II independently is utilized as the constant source of transfer. An analysis method is designed as follows. The schematic diagram of this method in a two-task environment is shown in Fig. 1. Assume that $task_1$ is the source task, which provides knowledge, and its corresponding sub-population is represented in yellow. $Task_2$ is the target task, which receives knowledge, and the corresponding sub-population is represented by purple. First, keep the decision vectors of all solutions optimized by NSGA-II of $task_1$ and $task_2$ every certain number of generations. It is set as ten generations in this paper. Second, $task_1$ uses the population in the final generation possessing the best convergence and diversity to transfer the knowledge to the populations preserved every fixed generation of $task_2$. The transfer strategy is simulated binary crossover (SBX) operator [37] which is as the same as in MOMFEA. After the information transfer, evaluate the generated population to judge the effect of the information transfer. The red square represents the population generated by information transfer on $task_2$. The effectiveness of the information transfer can be analyzed by comparing the performance of the sub-populations on the target task before and after the transfer, such as the corresponding purple and red squares.

The benefits of this analysis method can be summarized in the following two points. First, the transfer information from the source task is certain. Second, the transfer information is taken from the final population that has converged in the source task, which can avoid the influence of noise. However, the following points need to be noted. First, the information from the source task is settled which has fine convergence and diversity. The purpose of the experiment is only to expound the conceivable maximum benefit of the information transfers. This is slightly different from the real iteration of the EMT algorithms. Secondly, the process of information transfer is discrete rather than continuous; that is, the transfer occurs only at regular intervals. Thirdly, after the transfer, the generated population does not continue to evolve. There is no process of merging and selection of the population. The performance indicators such as inverted generational distance (IGD) [42] will be directly calculated at the generated population. Furthermore, the performance indicator calculation is not based on the

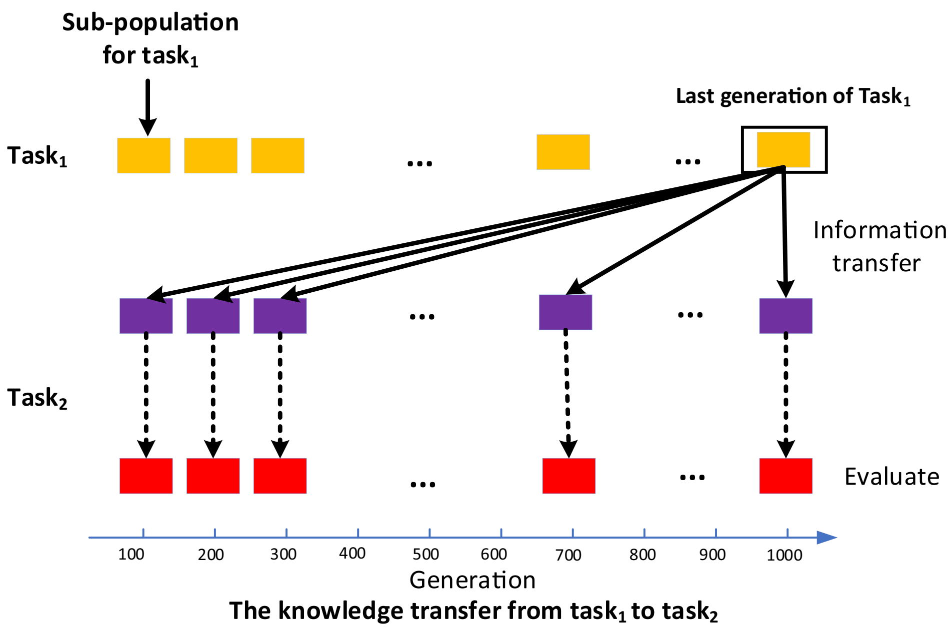

Fig. 1. Illustration of the knowledge transfer analysis method.

splendid individuals selected from the union population. In this section, only the occurrence cause and impact of negative transfer are considered. The specific strategies to reduce negative transfer are reflected in Section 3. Finally, in summary, through the information transfer analysis method, when a negative transfer occurs, the value of IGD, which is a commonly used multi-objective quality indicator, will rise. When a positive transfer occurs, the IGD value will fall.

By using the information transfer analysis model, Fig. 2 shows the iterations of the target population after it receives the information of the final source population, which evolved by 1000 generations. The SBX applied randomly with a fixed *rmp* was used as the information transfer method [16]. In Fig. 2 (a), the target task CIHS2 can quickly converge within 200 generations after receiving information from the final population of CIHS1. This indicates that the information transferred into CIHS2 played a positive guiding role. In Fig. 2 (b), the target task NILS2 not only fails to promote convergence after receiving the information from the final population of NILS1 but worsens the performance. This phenomenon is called a negative transfer.

The occurrence of a negative transfer is often caused by ignoring the differences between simultaneously optimized tasks, neglecting the improper information transfer timing, and overlooking the sensitive probability of an information transfer, which are the three most important factors that affect the information transfer efficiency.

2.3.1. Ignoring the differences between tasks

When optimizing multiple MOPs simultaneously, the overall convergence performance dividend of the algorithm brought about by an information transfer is positively correlated with the intersection of the PSs and the similarity of the fitness landscapes [35]. Using the EMT algorithm to optimize the two tasks with a high similarity will promote the convergence of both. Otherwise, it will harm the overall performance. From the perspective of the biome symbiosis [36], the ideal form of symbiosis is mutualism, where different species cooperate and benefit each other. The second form of symbiosis is parasitism. The two species living together have both positive and negative effects. The third way is competition, in which different species compete with and harm each other.

In the EMT area, the simultaneous task relationships can also be interpreted as mutualism, parasitism, and competition [24]. In Fig. 3(a) and (d), the relationship between the two tasks is mutualism. The PSs of the two are the same. The first decision variables x_1 of the two tasks are both diversity-related variables [38], their second decision variables x_2 converge to 0.4, and the fitness landscapes of both are similar. Therefore, information transfer can promote the convergence of both. However, for parasitism, as shown in Fig. 3(b) and (e), although x_1 of the two tasks are both diversity-related and x_2 of the two tasks are both convergence-related, the optimal value of task 1 only partially overlaps with that of task 2. When x_2 of all individuals converges to 0.4, it is advantageous for task 2 but disadvantageous for task 1. There is a significant difference between the fitness landscapes of both. The PF of task 1 is concave, whereas that of task 2 is convex. As for the competition relationship, as shown in Fig. 3(c) and (f), the characteristics of x_1 and x_2 of the two tasks are entirely different. When a specific dimension of one task is expected to converge to a point, the dimension of another task is expected to be dispersed as much as possible. The PFs of the two problems are different. Task 1 is a three-objective problem, whereas task 2 is a two-objective problem, and the fitness landscapes of the two are entirely different. When the two tasks are optimized simultaneously, an improvement in one task leads to the deterioration of another task. Most EMT algorithms ignore the relationship between tasks and apply the transfer strategy in mutualism to the two competing tasks, leading to a negative transfer during the iterations.

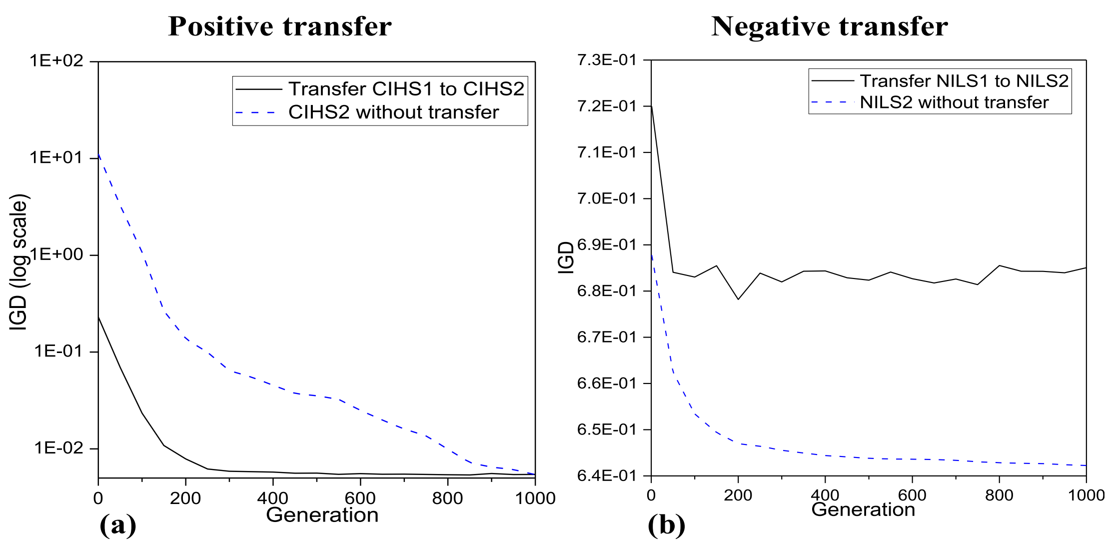

Fig. 2. Illustration of positive and negative transfers between tasks.

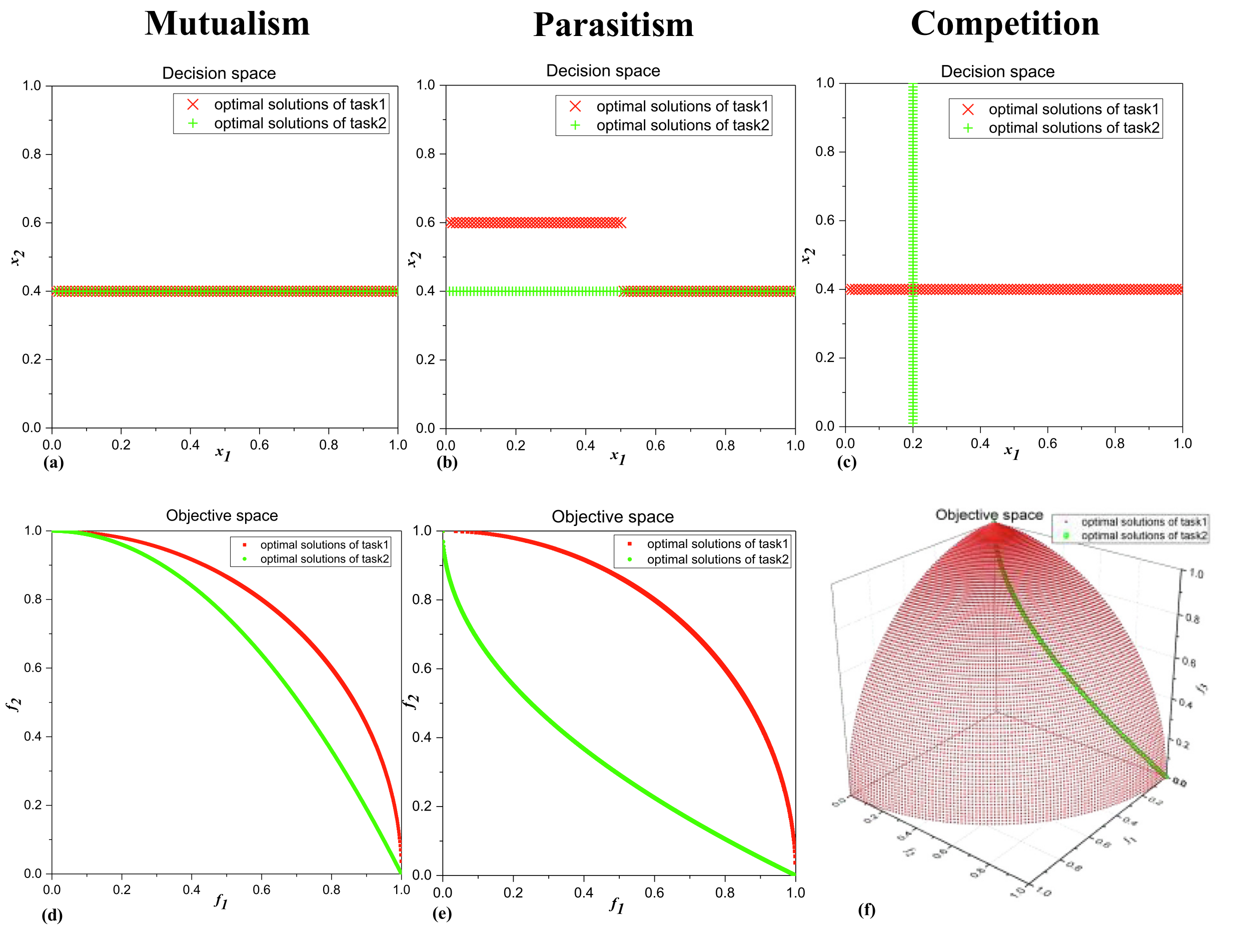

Fig. 3. Illustration of mutualism, parasitism, and competition between tasks.

In this study, the proposed horizontal cultural transmission strategy applies the decision space mapping strategy, which can map the decision space of the source task to the target task to reduce differences between tasks. The proposed adaptive information transfer strategy dynamically adjusts the probability of the information transfer. When the two tasks are not similar or a negative transfer is possible, the frequency of an information transfer is adaptively reduced.

2.3.2. Improper information transfer timing

The timing of an information transfer between tasks also plays an essential role in the information transfer efficiency. Most of the current EMT algorithms ignore the timing of the information transfer and apply the same *rmp* during the entire process of the population evolution, making the information transfer effective during the early stage but harmful during the later stage. Owing to the difficulty of the task, the convergence speed of the task is different. When the solution of the source task gradually approaches the optimal solution of the target task, it often brings about a positive transfer. Otherwise, it will damage the convergence of the target task. For example, in Fig. 4, the solid red line and the blue arrow line represent the approximate iterative trend of an individual when the target and source tasks run independently, where $\mathbf{a_0}$ and $\mathbf{b_0}$ represent the initial state of solutions **a** and **b**, respectively; $\mathbf{a_{best}}$ and $\mathbf{b_{best}}$ are the final states of the solutions; and $\mathbf{b_1}$, $\mathbf{b_2}$, and $\mathbf{b_3}$ are the intermediate states from $\mathbf{b_0}$ to $\mathbf{b_{best}}$. When $\mathbf{a_0}$ and $\mathbf{b_1}$ exchange information, the generated offspring may be approximately $\mathbf{b_1}$. It helps $\mathbf{a_0}$ move directly toward the vicinity of $\mathbf{a_{best}}$ without an iteration. This is the appropriate time for an information transfer. When $\mathbf{a_0}$ and $\mathbf{b_3}$ exchange information, the offspring generated may be around $\mathbf{b_3}$, which is farther away from $\mathbf{a_{best}}$ than $\mathbf{b_1}$, and thus this is an inappropriate time for a transfer.

When the source and target tasks have similar evolutionary trends, any moment in the optimization process is a suitable transfer time. Conversely, when two tasks have different evolutionary trends, the appropriate transfer timing must be carefully selected. By using the information transfer analysis model, Fig. 5 shows the results obtained when saving the approximate PS after the source task has evolved separately for 1000 generations and conducting an information transfer [16] with the target task. An information transfer adopts binary tournament selection and an SBX crossover operator [37]. As shown in Fig. 5(a), PIHS2 and PIHS1 exhibited similar evolutionary trends. Therefore, after PIHS1 adopts the information from the final

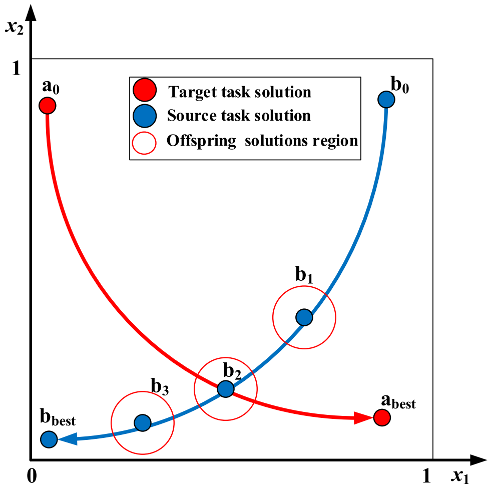

Fig. 4. Illustration of the relationship between the transfer timing and evolutionary trend.

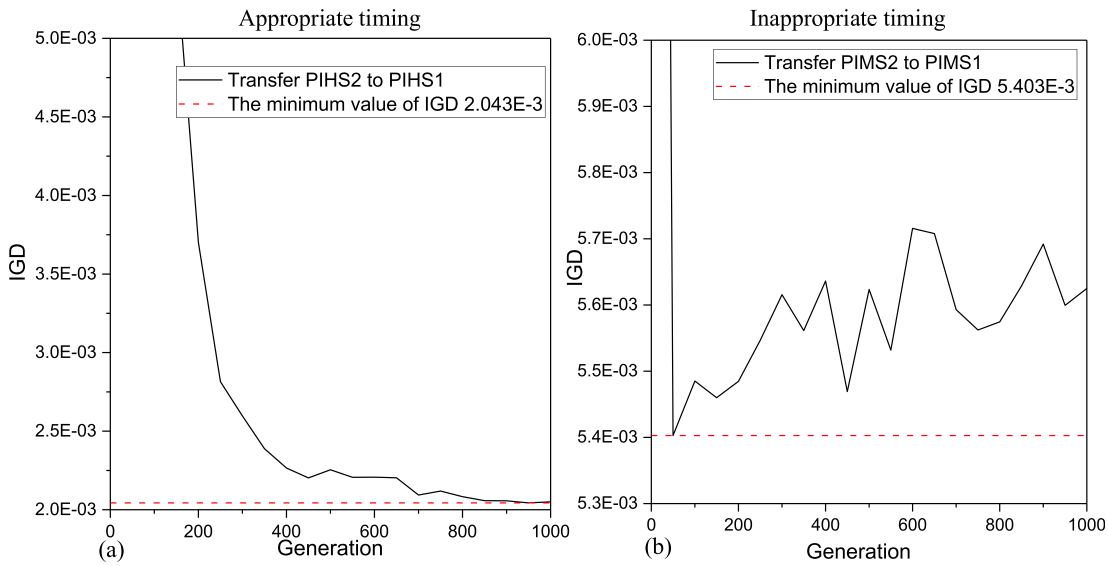

Fig. 5. Illustration of the right and inappropriate timing of an information transfer.

result of PIHS2, it can approach the true PF. However, in Fig. 5(b), PIMS2 and PIMS1 have different evolutionary trends, and thus PIMS1 receives the information from the final result of PIMS2. The performance in PIMS1 will first quickly improve and then gradually deteriorate. Given that an information transfer can only produce positive effects within a period of iteration, the timing of the transfer should be controlled. The probability of an information transfer at the appropriate time should be increased and reduced at the inappropriate time.

In this study, the proposed adaptive information transfer strategy has a reward and penalty mechanism. When the evolution using the source task information has a higher success rate than that using the target task information, the *rmp* will be rewarded. This process can be superimposed. When *rmp* increases to close to 1, this period is considered as the right transfer timing. By contrast, the *rmp* is continuously punished. When *rmp* is reduced to close to zero, this may be an inappropriate time, and the information transfer will be stopped.

2.3.3. Sensitive information transfer probability

In EMT algorithms, the probability of an information transfer represents the probability that an individual receives the information of the source task during one evolution. In the MFEA framework, it is described as *rmp*. The larger the value of *rmp* that is applied, the greater the probability that an individual will receive information from the source population.

Conversely, more evolutionary information is obtained from the target task. When the relationship between the two tasks is mutualism, a larger *rmp* can make the two tasks frequently exchange information to promote mutual convergence. However, when the relationship is competitive, a larger *rmp* will cause the two to hinder each other. As shown in Fig. 6(a), (b), for two highly similar problems, CIHS1 and CIHS2 [35], as the value of *rmp* increases, the exchange of information between the two will increase, making them converge more rapidly. For two competing tasks, such as NILS1 and NILS2, with an increase in *rmp*, the performance of the algorithm worsens, as shown in Fig. 6(c), (d).

In real optimization problems, the relationship between simultaneously optimized tasks is unknown in advance. Most of the current algorithms do not consider the relationship between tasks, and all tasks are artificially assigned with a fixed *rmp* set. This restricts the EMT algorithms from exerting the advantages of an information transfer on two similar tasks and leads to a negative transfer on competing tasks, which reduces the performance of the algorithm.

In this study, the proposed algorithm allocates evolutionary resources in the initialization phase evenly. According to the rules of a reward and penalty during the evolution process, *rmp* can be dynamically changed. The value of *rmp* can be increased as much as possible on highly similar tasks and reduced in a timely manner in competing tasks. The value of *rmp* will increase at the appropriate time for a transfer and decrease appropriately at an inappropriate time.

3. Proposed algorithm

In this paper, a cultural transmission-based multi-objective evolution strategy algorithm is proposed for solving MOMTO problems, which can address the challenges listed in Section 2.3 efficiently and effectively.

3.1. Elite guided variation strategy

The evolution strategy is a heuristic algorithm that imitates the biological evolution. Unlike genetic algorithms, more emphasis is placed on behavior changes at the individual level. The individual only produces offspring through mutations and only compares the advantages and disadvantages with its parents. Therefore, if an individual can absorb superb information from the population, it can converge quickly. Modern cultural evolution theory [39] believes that the thoughts and speech of high-ranking people will substantially impact the entire population. Therefore, an elite-guided variation strategy in which individuals are impacted culturally from elite individuals, such as individuals on the current PF, is proposed in this paper. By accepting genetic information from excellent individuals, poor individuals can converge quickly. Fig. 7 shows the performance of an individual in the objective space after conducting the elite-guided variation strategy in all dimensions of the decision variables. The green inferior solutions crossover with the red non-dominated solutions by an operator such as SBX [37], the offspring will be around the current PF.

This method can continuously advance inferior solutions toward the PF and significantly improve the convergence performance of the algorithm. The pseudo-code of the elite-guided variation strategy is presented in Algorithm 1.

Algorithm 1
Elite-guided variation strategy

Input: $\mathbf{p}$: the parent individual, d: the index of the decision variable, *POS*: Pareto optimal solution set in current population.
Output: $\mathbf{o}$: the offspring individual.
1. $\mathbf{o} = \mathbf{p}$.
2. $\mathbf{e} = random(POS)$.
3. $\mathbf{o}^d = SBX\ (\mathbf{p}^d, \mathbf{e}^d)$.
4. $\mathbf{o}$.skill factor = $\mathbf{p}$.skill factor.

A characteristic of the evolution strategy is to change only a single dimension of the decision variables each time. In the proposed elite-guided variation strategy, offspring $\mathbf{o}$ will first inherit all genes from parent $\mathbf{p}$. Then, individual $\mathbf{e}$ is chosen at random from the current PF, and will hybridize with $\mathbf{p}$ on the *dth* dimension and assign the result to the *dth* dimension of $\mathbf{o}$. Finally, $\mathbf{o}$ inherits most of the genes of $\mathbf{p}$, obtains the gene of the *dth* dimension of the non-dominated solutions, and inherits the skill factor of parent $\mathbf{p}$.

3.2. Horizontal cultural transmission strategy

Inspired by modern cultural evolution theory [39], people from different cultures will collaborate and share their thoughts and experiences. Therefore, a horizontal cultural transmission strategy is proposed to transfer information between various tasks and promote the overall convergence efficiency by using the implicit complementarity between tasks and sharing the knowledge that other tasks have learned. The pseudo-code is presented in Algorithm 2.

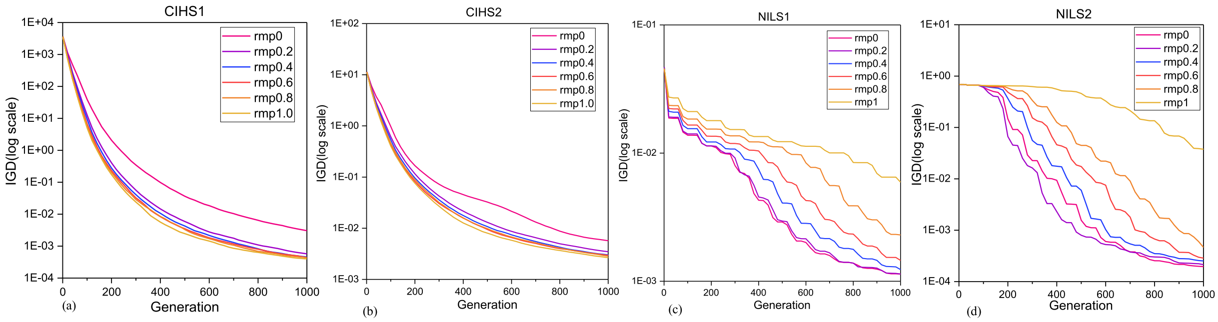

Fig. 6. Different *rmp* applied to CIHS and NILS benchmark problems.

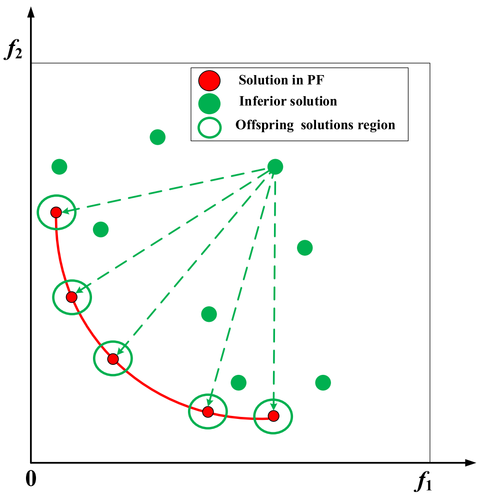

Fig. 7. Performance of an individual in objective space after applying the elite-guided variation strategy in all dimensions of the decision variables.

Algorithm 2
Horizontal cultural transmission strategy

Input: $\mathbf{p}$: the parent individual, S: the population of the source task, d: the index of the decision variable, $DimS$: the number of dimensions of the decision vector of source task.
Output: $\mathbf{o}$: the offspring individual.
1. $\mathbf{o} = \mathbf{p}$.
2. $\mathbf{r} = random(S)$.
3. $randdim = random(DimS)$.
4. $\mathbf{r}^{transfer}$ = ***decision space mapping*** $(\mathbf{r}^{randdim})$.
5. $\mathbf{o}^d = SBX\ (\mathbf{p}^d, \mathbf{r}^{ransfer})$.
6. $\mathbf{o}$.skill factor = $\mathbf{p}$.skill factor.

The horizontal cultural transmission strategy is the main component of the information transfer, and the motivation is to learn information from other tasks to improve the overall convergence performance of simultaneous optimization tasks. Compared with the elite-guided variation strategy, the same point is that they both apply SBX as the crossover operator. Current research has shown that the application of the SBX operator in the same task can improve the exploitation ability, and the SBX operator in different tasks can improve the exploration ability of the algorithm [34]. Nevertheless, the differences between the horizontal cultural transmission and elite-guided variation strategy lie in the following points. First, the objects for exchanging genetic genes are different. The crossover object of the elite-guided variation strategy comes from the PF of the target task. In the horizontal cultural transmission strategy, the individual is derived from a solution randomly selected in the source task. Second, the dimensions of the gene exchange are different. In the elite-guided variation strategy, a gene exchange occurs in the same dimension. In the horizontal cultural transmission strategy, the individual exchanges a gene with a random dimension of the individual in the source task. Finally, as the most critical difference, the decision space mapping strategy is used in the horizontal cultural transmission strategy, which can map the decision variables of the source task to the target task. These three differences establish the effectiveness of the information transfer.

There are three reasons why the horizontal cultural transmission strategy randomly selects solutions from the source task to conduct a gene exchange in a random dimension. First, the PSs and fitness landscapes of different tasks are different. The values of the decision variables corresponding to the current PF on the source task are generally far from the PS of the target task. Second, the number of decision variables is different, making it impossible to transfer the decision variables of the source task and the target task in a one-to-one correspondence. Finally, the characteristics of the corresponding decision variables for different tasks are diverse. For example, the convergence-related variable [38] hopes that all solutions can converge to one point in this dimension. By contrast, the diversity-related variable expects that all solutions in this dimension can be dispersed from each other. To reduce the difference in the decision space between the source and target tasks, a decision space mapping strategy is proposed, which can project the decision variables from the source task to the target task by averaging, as shown in Eq. (3).

$$\mathbf{r}^{transfer} = \frac{mean\left(P^{d}\right) + \varepsilon}{mean(S^{randim}) + \varepsilon} \times \mathbf{r}^{randim} \tag{3}$$

where $\mathbf{r}^{randim}$ denotes the *randdimth* dimension of the individual $\mathbf{r}$ from the source task, $mean(S^{randim})$ is the mean value of all individuals from the source task in the *randimth* dimension, and $mean\left(P^{d}\right)$ indicates the mean value of all individuals from the target task in the *dth* dimension. In this way, the information transferred from the source task can be zoomed into the boundary of the target task. The spatial characteristics of the source task can be maintained to minimize the difference between the target and source tasks in the decision space.

3.3. Adaptive information transfer strategy

The proposed adaptive information transfer strategy dynamically changes the probability of information transfer *rmp* by comparing the mutation success rate of the target task itself and the success rate of the information transfer to change the allocation of the evolutionary resources for future generations. In the initialization phase, *rmp* is set to 0.5 to allocate the evolutionary resources evenly. The calculation formulas for *srh* and *sre* are given in Eq. (4).

$$sre = \frac{N_{se}}{Dim}$$

$$srh = \frac{N_{sh}}{Dim} \tag{4}$$

where N_{se} and N_{sh} represent the number of offspring better than the parent obtained by the elite-guided variation strategy and horizontal cultural transmission strategy in all dimensions of an individual, respectively, and *Dim* represents the number of dimensions of the target task. The proposed adaptive information transfer strategy can solve the problems raised in Section 2.3, adaptively adjust the probability of information transfer to reduce the negative transfer caused by task differences, and adaptively adjust the information transfer timing. The pseudo-code of adaptive information transfer strategy is shown in Algorithm 3.

Algorithm 3
Adaptive information transfer strategy

Input: N_h: the number of individuals generated by horizontal cultural transmission strategy, *rmp*: the information transfer probability of the target task, *sre*: the success rate of elite-guided variation strategy, *srh*: the success rate of horizontal cultural transmission strategy.
Output: *rmp*: the information transfer probability of the target task.

```
If (N_h == 0) then
  rmp = rmp + (1-sre).
Else
  If (srh > sre) then
    rmp = rmp + srh.
    If (rmp > 0.5) then
      rmp = 0.5.
    End if
  Else
    rmp = max (rmp – (1 - srh), 0).
  End if
End if
```

The adaptive information transfer strategy is based mainly on three mechanisms. The first is a reward mechanism. As shown in line 5, when the success rate of an information transfer is greater than the success rate of the variation, it indicates that the information transfer is beneficial. The probability of an information transfer increases. The second is a penalty mechanism, as shown in line 10. Here, when the transfer success rate of the target task information is less than the variation success rate, it indicates that the dividends brought about by the current information transfer are not as great as the mutation. The evolutionary resources used for its own mutation should be increased such that the population can accept more information from its own tasks. The last is the evolutionary resource redistribution mechanism used to solve the unbalanced distribution of the evolutionary resource problem. When the probability of the information transfer decreases, fewer evolutionary resources are allocated to the information transfer strategy. Subsequently, the number of offspring generated by the horizontal cultural transmission strategy is reduced, and the probability of the successful transfer also decreases. Owing to this positive feedback mechanism, all resources are obtained by the elite-guided variation strategy. Conversely, when the success rate of the information transfer method becomes larger, more resources will flock to the information transfer method, and thus fewer offspring will be obtained by the elite-guided variation strategy, causing the success rate of the elite-guided variation strategy to become smaller, and ultimately, all solutions will be obtained through the information transfer method. This feedback is irreversible. Once started, all of the evolutionary resources flood into one evolutionary approach. The final population can only be obtained by either the elite-guided variation strategy or horizontal cultural transmission strategy, and the adaptive transfer method will fail. The unbalanced distribution of evolutionary resources is shown in Fig. 8.

In Fig. 8(a), during the early stage of the evolution, *sre* is always greater than *srh*, resulting in a continuous decrease in *rmp*. After it is reduced to zero, the evolutionary resources are no longer allocated to the horizontal cultural transmission strategy. The information transfer method fails in the subsequent evolutionary process, and the population can only accept information from the target task. In Fig. 8(b), *srh* surpasses *sre*, leading to a continuous increase in *rmp*. Eventually, all evolutionary resources are plundered by the horizontal cultural transmission strategy, and the elite-guided variation strategy fails. The proposed evolutionary resource redistribution mechanism can solve the above two problems, which is divided into two parts: a minimum guarantee strategy and a restart strategy. The minimum guarantee strategy means that if there is not one individual in the population generated by the horizontal cultural transmission strategy, *rmp* will increase such that the population can have the opportunity to learn potentially helpful knowledge from the source task, as shown in line 2. In the restart strategy, if the success rate of the horizontal cultural transmission strategy is too high and *rmp* continues to grow such that all evolutionary resources will be plundered, *rmp* is controlled at a threshold of 0.5, as shown in line 7. A threshold

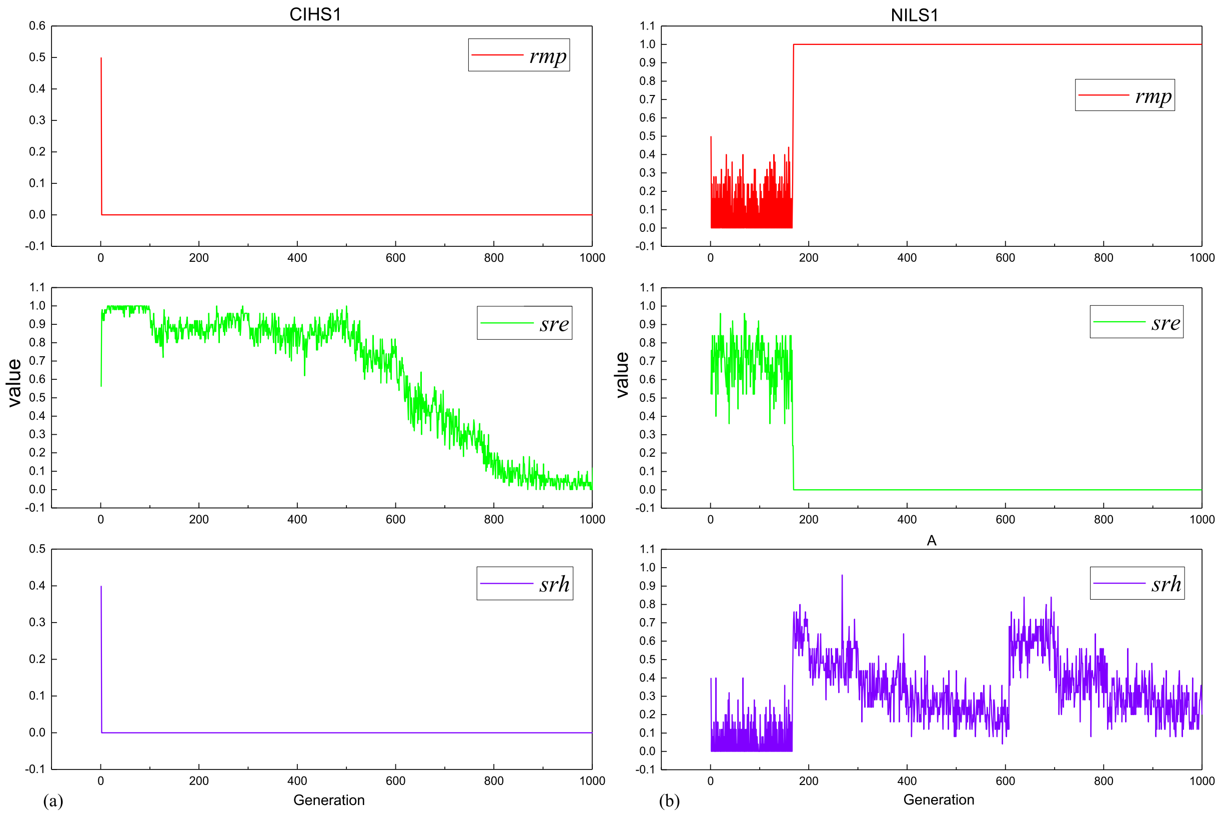

Fig. 8. Illustration of the unbalanced allocation of evolutionary resources.

of 0.5 means that the offspring will use the guided variation strategy and the horizontal cultural transmission strategy to generate new solutions with the same probability.

Although MPEF [24] proposed a similar evolutionary resource allocation strategy, the proposed adaptive information transfer strategy is based on a completely different idea. The difference between them lies in the following three points: First, the proposed adaptive information transfer strategy does not require additional artificially set parameters, does not need a threshold to start this strategy, and does not need to set the allocation ratio parameter. Second, MPEF is a single-objective algorithm that uses the objective function value as the fitness value. The proposed CT-EMT-MOES is suitable for multi-objective optimization and uses the dominance relationship between the parent and offspring to determine whether the evolution is successful. Finally, MPEF does not have a mechanism for redistributing evolutionary resources, leading to an unbalanced evolutionary resource problem.

3.4. Complete proposed algorithm

Multi-objective EMT algorithms need to have three capabilities: convergence, diversity, and information transferability. The proposed CT-EMT-MOES is based on the vector equilibrium-based evolution strategy (MaOES) [3], which is ensured by comparing the dominance relationship between the offspring and its parent, and where the maximum extension distance (*MED*) is used to maintain the diversity of the population. When the parent and offspring are in a non-dominated relationship, the pros and cons of the solution are judged by the number of *Dom*, which indicates the number of solutions that can dominate it and the *MED*. The horizontal cultural transmission and adaptive information transfer strategies are proposed to endow the algorithm with information transferability. Therefore, as a superior aspect of MaOES, the algorithm can cover the real PF using only a few solutions. The formal representation of the *MED* is as Eq. (5).

$$
\begin{aligned}
&MED(\mathbf{x}_i) = TotalDist(\mathbf{x}_i) \times NearDist(\mathbf{x}_i) \\
&TotalDist(\mathbf{x}_i) = \sum_{j=1}^{N} \sum\nolimits_{m=1}^{M} \sqrt[2]{(f_m^i - f_m^j)^2} \\
&NearDist(\mathbf{x}_i) = \min_{i,j \neq i} \sum\nolimits_{m=1}^{M} \sqrt[2]{(f_m^i - f_m^j)^2}
\end{aligned}
\tag{5}
$$

where N represents the number of individuals in the sub-population, and M represents the number of objective functions in the target task. The $TotalDist(\mathbf{x}_i)$ calculates the sum of the Manhattan distance in the objective space between $\mathbf{x}_i$ and other solutions in the sub-population. A larger $TotalDist(\mathbf{x}_i)$ indicates that the solution is farther away from the other solutions in the objective space. The $NearDist(\mathbf{x}_i)$ calculates the minimum Manhattan distance in the objective space between $\mathbf{x}_i$ and other solutions in the sub-population. In addition, *MED* is committed to extending the boundaries of the current population in the objective space. A larger *MED* indicates that the solution has a better individual diversity.

The overall framework of the proposed CT-EMT-MOES is summarized in Algorithm 4. First, on line 1, the entire population is initialized in the unified decision space. Then, the entire population is divided evenly into K sub-populations, each of which corresponds to a separate task on line 2. On line 3, each individual in the sub-population is assigned a corresponding skill factor and evaluated. The value of *rmp* is initialized to 0.5 to allocate the evolutionary resources evenly to the two strategies on line 4. In the case that the algorithm termination condition is not reached, each task P_k is traversed and is considered as the current target task. Next, a random integer j which represents the index of the source task is generated. Whereafter, traverse and evolve each individual $\mathbf{x}_i^{(k)}$ in P_k. Parameters utilized to trade off two evolutionary strategies N_h, N_{se}, and N_{sh} are initialized as zero. N_h represents the number of individuals generated by the horizontal cultural transmission strategy, N_{se} denotes the number of individuals generated by elite-guided variation strategy surpass their parents, and N_{sh} expresses the number of individuals generated by the horizontal cultural transmission strategy transcends their parents. Then, the non-dominated solutions in the current sub-population P_k are collected into temporary population POS_k. Due to the evolutionary characteristics of MaOES [3], the size of POS_k will never exceed the sub-population size. Subsequently, for each decision variable of the kth task, if the random number *rand* between zero to one is larger than *rmp*, the elite-guided variation strategy is used to mutate the decision variable, and the flag *Istransfer* is set to False. Otherwise, the horizontal cultural transmission strategy is used as the evolution operator, and the *Istransfer* flag is set to True. Then, offspring $\mathbf{o}$ is mutated by the precision-controllable mutation operator [3] proposed in MaOES. Following the update strategy of the MaOES, if the offspring can dominate the parent, the offspring is accepted. Otherwise, if the two are non-dominated and $Dom(\mathbf{o}) < Dom(\mathbf{x}_i^{(k)})$, where $Dom(\mathbf{o})$ indicates the number of individuals that can dominate the $\mathbf{o}$ in the sub-population, the parent is replaced by the offspring. If the *Dom* of both is the same, we consider the *MED* values of the individuals. If an offspring can replace its parent, this mutation is considered a successful mutation, and the corresponding number of successful mutations is increased by one. When all dimensions of an individual are mutated, the success rates of the two mutation strategies, *sre* and *srh*, are updated based on the number of successful mutations. The value of *rmp* is updated using an adaptive information transfer strategy. Finally, if the stop criterion is met, the final population is output as the result of all tasks.

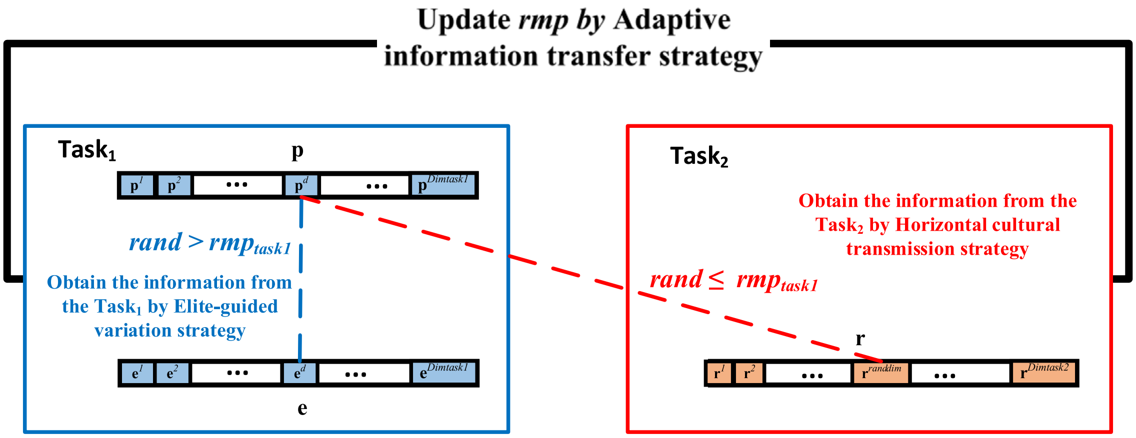

Fig. 9. An example of the proposed CT-EMT-MOES for an individual p in a two-task environment.

As an illustrating example, Fig. 9 shows the evolutionary process for an individual **p** in a two-task environment. Individual **p** serves for task1, the target task, and task2 is considered the source task. In the proposed CT-EMT-MOES, each dimension of the decision variables of a single individual will be considered in turn. Assume that *Dimtask1* and *Dimtask2* are the total number of decision variable dimensions of task1 and task2, respectively. First, when evolving $\mathbf{p}^d$, the *dth* dimension of the individual **p**, a random decimal *rand* between zero to one will be generated. If *rand* is greater than the rmp_{task1}, which is the current threshold of the cross-task information transfer of task1, the elite-guided variation strategy is activated. Specifically, the crossover operator is conducted between **p** and **e** in the *dth* dimension, where **e** is a random individual in the current PF in task1. Then, whether to accept this evolution is judged by comparing the performance of **p** on task1 before and after this evolution. If the evolution can make **p** perform better on task1, $\mathbf{p}^d$ will accept this evolution. Through the crossover operation, the excellent genes on **e** will be passed on to **p**. If *rand* is less than or equal to the rmp_{task1}, the horizontal cultural transmission strategy is activated. Specifically, the crossover operator is conducted between the *dth* dimension of **p** and the *randdimth* dimension of **r**, where **r** is a random individual in the source task and *randdim* is a random dimension of **r**. If the evolution can make **p** perform better on task1, $\mathbf{p}^d$ will accept this evolution. Through the crossover operation, the genes on **r** will be passed on to **p**. The crossover operator is the same as in MFEA [11], which applies SBX. After all the dimensions of individual **p** have evolved, the proposed adaptive information transfer strategy is activated to update rmp_{task1} adaptively through comparing the success rate of the two strategies.

3.5. Complexity analysis

In this section, the computational complexity of one generation of the proposed CT-EMT-MOES is discussed. Suppose D denotes the total number of dimensions of the unified decision space, K indicates the total number of tasks, N indicates the total number of all individuals, N_k expresses the size of the kth sub-population, and M_k indicates the number of objective functions in the kth task. The time complexity of the elite-guided variation strategy, adaptive information transfer strategy, and mutation operator are all $O(1)$. The time complexity of the horizontal cultural transmission strategy and the Pareto optimal solution set operators obtained are $O(N_k)$ and $O(M_k N_k^2)$, respectively. The time complexity when calculating *Dom* and *MED* is $O(M_k N_k)$. Overall, the total computational complexity of CT-EMT-MOES is $K \times N_k \times (O(M_k N_k^2) + Dim_k \times (O(1) + O(N_k) + O(1) + O(M_k N_k) + O(M_k N_k)) + O(1))$. According to the operation rules of symbol O, the time complexity of the proposed CT-EMT-MOES can be simplified as $O(MN^3)$.

Algorithm 4
The overall framework of CT-EMT-MOES

Input: N, the total population size; K, the number of tasks.
Output: the final solution set.
1. Initialize the total population.
2. Evenly distribute the entire population to K sub-populations P_k for task T_k (k = 1, 2, 3…K).
3. Assign skill factors to the individuals and evaluate them according to their skill factors.
4. Set *rmp* in all tasks to 0.5.
5. **While** stop criterion is not met **do**
6. **For each** T_k

$j = random(K)$.
For each $\mathbf{x}_i^{(k)}$ in P_k
$N_h = 0$, $N_{se} = 0$, $N_{sh} = 0$.
POS_k = Pareto optimal solution set in current P_k.
For $d = 1$ **to** $DimP_k$
If ($rand > rmp$) **then**
Istransfer = False.
$\mathbf{o}$ = ***Elite-guided variation strategy*** ($\mathbf{x}_i^{(k)}$, d, POS_k).
Else
Istransfer = True, $N_h = N_h + 1$.
$\mathbf{o}$ = ***Horizontal cultural transmission strategy*** $(\mathbf{x}_i^{(k)}, P_j, d, DimP_j)$.
End if
$\mathbf{o}$ = ***Mutation*** ($\mathbf{o}$).
set $\mathbf{o}$. skill factor = $\mathbf{x}_i^{(k)}$skill factor and evaluate $\mathbf{o}$ at task k.
If $\mathbf{o} \prec \mathbf{x}_i^{(k)}$ **then**
$\mathbf{x}_i^{(k)} = \mathbf{o}$.
If (*Istransfer* == True) **then**
$N_{sh} = N_{sh} + 1$.
Else
$N_{se} = N_{sE+}1$.
End if
Elseif $\mathbf{o}$ and $\mathbf{x}_i^{(k)}$ are non-dominated with each other **then**
If $Dom(\mathbf{o}) < Dom(\mathbf{x}_i^{(k)})$ **then**
$\mathbf{x}_i^{(k)} = \mathbf{o}$.
If (*Istransfer* == True) **then**
$N_{sh} = N_{sh} + 1$.
Else
$N_{se} = N_{sE+}1$.
End if
Elseif $Dom(\mathbf{o}) = Dom(\mathbf{x}_i^{(k)})$ and $MED(\mathbf{o}) > MED(\mathbf{x}_i^{(k)})$ **then**
$\mathbf{x}_i^{(k)} = \mathbf{o}$.
If (*Istransfer* == True) **then**
$N_{sh} = N_{sh} + 1$.
Else
$N_{se} = N_{sE+}1$.
End if
End if
End if
End for
$sre = N_{se}/DimP_k$, $srh = N_{sh}/DimP_k$.
Adaptive information transfer strategy (N_h, rmp, sre, srh).
End for
End for
End while

4. Experiments

In this section, the proposed CT-EMT-MOES is compared with the state-of-the-art EMT algorithms MOMFEA [16], MOMFEA-II [26], MaTDE [20], MFEA-AKT [25], MTEA-AD [27], and the well-known MOEA NSGA-II [2]. The performance of CT-EMT-MOES was comprehensively evaluated using the classical MOMTO benchmark test suite [35] and the complex MOMTO benchmark test suite [40].

4.1. Test suite introduction

In MTO problems, the similarity of the fitness landscapes and the degree of intersection of the optimal solutions are the two foremost factors influencing the effectiveness of the genetic information transfer between tasks. If the corresponding dimensions of the optimal solutions of different tasks are closer, the genetic information transfer between tasks is more conducive to the optimization. Similarly, the more similar the fitness landscapes of the optimization functions of different tasks are, the more the knowledge that an individual learns from the source task can help indirectly optimize the target task. In MOPs, as the necessary and sufficient condition for obtaining the PS, the constructor function $q(\mathbf{x})$ of the objective functions takes the global optimum. Therefore, the two factors that affect the transfer efficiency in MOMTO can be transformed into the relationship $q(\mathbf{x})$ in different tasks. According to the intersection degree of the global optimum, the classical MOMTO benchmark test problems are designed into the complete intersection (CI), partial intersection (PI), and no intersection (NI). According to the similarity of the fitness landscape, the classical MOMTO benchmark test problems can be divided into three classes: high similarity (HS), medium similarity (MS), and low similarity (LS). Based on the combination of the above two classification strategies, the classical MOMTO benchmark test suite contains nine continuous multi-objective sets of problems. Detailed information on the classical MOMTO benchmark problems can be found in [35]. The complex MOMTO benchmark test suite, CPLX, was introduced at the IEEE CEC 2020 Competition on Evolutionary Multitask Optimization [40], and is more complicated than the classical test suite. Its sub-problem was designed according to [41].

4.2. Compared algorithms

The proposed CT-EMT-MOES is compared with five state-of-the-art multi-objective EMT algorithms: MOMFEA [16], MOMFEA-II [26], MaTDE [20], MFEA-AKT [25], MTEA-AD [27], and a classic MOEA, NSGA-II [2]. MOMFEA [16] is the earliest and most classic multi-objective EMT algorithm and can be regarded as the benchmark of the multi-objective EMT algorithm. The well-known NSGA-II [2] was applied as the evolutionary engine for MOMFEA to process MOMTO problems. Comparing the performances of MOMFEA and NSGA-II, the advantages of the multi-objective EMT algorithm are demonstrated over traditional MOEA, and thus NSGA-II is also included as a comparison algorithm. MOMFEA-II [26], MFEA-AKT [25], and MTEA-AD [27], like the proposed CT-EMT-MOES, can be classified as approaches based on improving knowledge transfer strategy. MOMFEA-II [26] applied a data-driven online learning method to optimize the transfer intensity during the search process to solve MOMTO problems and reduce negative transfers. MFEA-AKT [25] adaptively configured the crossover operator for knowledge transfer between tasks based on the information collected in the online evolutionary search process to achieve robust and efficient multitasking on different optimization problems. MTEA-AD [27] constructed an anomaly detection model for each task. It can adaptively recognize valuable knowledge from multi-task environments with complex relationships through online learning of the relationships between the current task and individuals in other tasks. MaTDE [20] constructed an auxiliary task for each task through an adaptive selection mechanism combining the similarity measurement between tasks and the cumulative reward for knowledge transfer. It can transfer shared knowledge through implicit genetic crossover between two tasks and improve the search efficiency of the current task. All the algorithms are implemented using Jmetal 4.5.2, which is an object-oriented Java-based framework [44]. The algorithms are run on a PC with Intel Core i5-9400F CPU 2.90 GHz, and 16.00 GB of RAM.

4.3. Performance indicators

Three multi-objective quality indicators, that is the IGD [42], hypervolume (HV) [45] and additive epsilon (EPS_+) [46] are utilized to measure the performance of the proposed algorithm. For each benchmark problem, 10,000 points in the three-objective task and 1000 points in the two-objective task were sampled in the true PF [35,40] to calculate the metrics. The IGD indicator measures the distance between the true PF and the closest individual in the solutions obtained. This indicator can be expressed as in Eq. (6) where $Dist_i$ is the Euclidean distance between the ith solution in the true PF and the closest individual in the obtained solutions, and $|SP|$ is the number of sampling points. The lower the IGD value, the better the convergence and diversity of the population.

$$IGD = \frac{\left(\sum_{i=1}^{|SP|} Dist_i^2\right)^{1/2}}{|SP|} \tag{6}$$

HV calculates the volume covered by the members of obtained solution set P and reference point in the objective space. Specifically, for each solution in P, a hypercube v_i is established with the solution p_i and the reference point W as the diagonal corners of the hypercube. In [45], the reference point is constructed based on the vector of worst objective function values. Whereafter, a union of all hypercubes is obtained and the HV of the union hypercube is calculated by Eq. (7).

$$\mathrm{HV} = volume\left(\bigcup_{i=1}^{|P|} v_i\right) \tag{7}$$

HV appraises both convergence and diversity of solution set obtained. A higher HV value indicates a larger volume is dominated by the solution set obtained, thus corresponding to better performance.

The EPS_+ provides the minimum factor by which the obtained solution set P has to be translated in the M-dimensional objective space to weakly dominate the sampling points SP. EPS_+ is based on an additive factor. The mathematical expression of EPS_+ is shown in Eq. (8).

Table 1
Parameters setting for the experiments.

Parameter	NSGA-II	MOMFEA	MOMFEA-II	MaTDE	MFEA-AKT	MTEA-AD	CT-EMT-MOES
rmp	–	0.3	0.3	–	0.3	0.3	0.5
N	200	200	200	200	200	200	200
max *FEs*	200,000	200,000	200,000	200,000	200,000	200,000	200,000
p_c	0.9	0.9	0.9	–	0.9	0.9	0.9
η_c	20	20	20	–	20	20	20
p_m	$1/D$	$1/D$	$1/D$	–	$1/D$	$1/D$	–
η_m	20	20	20	–	20	20	–

$$EPS_+ = \max_{sp \in SP} \min_{p \in P} \max_{m \in \{1,\dots,M\}} (p_m - sp_m) \tag{8}$$

The smaller the value of EPS$_+$, which means the closer the obtained solution set *P* is to the true PF, the better the convergence of the algorithm. It is worth emphasizing that all evaluation indicators are implemented using built-in functions in Jmetal 4.5.2 [44].

4.4. Parameter settings

For a fair comparison, in MOMFEA [16], MOMFEA-II [26], MaTDE [20], MFEA-AKT [25], and MTEA-AD [27], the total population size is set to 200. However, in single-task MOEA NSGA-II [2], the population size of each task is set to 100. For the EMT algorithms, the maximum number of fitness evaluations is 200,000, whereas for conventional MOEAs, such as NSGA-II [2], the maximum number of evaluations for each task is set to 100,000. The initial random mating probability *rmp*, the total population size for all tasks *N*, the maximum number of evaluations max *FEs*, the crossover probability p_c, the distribution index of crossover η_c, the mutation probability p_m, and the distribution index of mutation η_m settings are listed in Table 1. The other parameter settings of the comparison algorithms are the same as those in the original papers.

4.5. Performance on classical MOMTO benchmark problems

The IGD, HV, and EPS$_+$ values of the classical MOMTO benchmark test suites are listed in Table 2, Table 3, and Table 4, respectively. The average values over 20 independent runs of each algorithm are demonstrated, and the best result for each sub-problem is marked in gray. In addition, the Wilcoxon rank-sum test at the 95% confidence level was applied for the experimental results to compare the proposed CT-EMT-MOES with other comparison algorithms, where significantly better, significantly worse, and not comparable are indicated using "+", "−", and "=", respectively. In the classical MOMTO benchmark test suites, the multi-objective EMT algorithm can defeat the classic single-task MOEA NSGA-II on most benchmark test problems. This is mainly due to the information transfer mechanism of the EMT algorithm, which indicates that the multi-

Table 2
Averaged value of IGD obtained by NSGA-II, MOMFEA, MOMFEA-II, MaTDE, MFEA-AKT, MTEA-AD, and CT-EMT-MOES on the classical MOMTO benchmark test suite.

Problem	Task	NSGA-II	MOMFEA	MOMFEA-II	MaTDE	MFEA-AKT	MTEA-AD	CT-EMT-MOES
CIHS	T1	2.03E-03(+)	3.64E-04(+)	4.69E-04(+)	2.05E-04(+)	2.55E-04(+)	1.91E-03(+)	1.88E-04
	T2	4.36E-03(+)	2.83E-03(+)	2.36E-03(+)	4.49E-04(+)	1.61E-03(+)	4.57E-03(+)	1.94E-04
CIMS	T1	1.23E-01(-)	6.64E-02(-)	1.76E-01(+)	1.42E-01(+)	1.89E-01(+)	8.75E-02(-)	1.33E-01
	T2	2.58E-02(+)	8.85E-03(-)	3.88E-04(-)	3.53E-04(-)	2.03E-04(-)	1.94E-04(-)	1.45E-02
CILS	T1	2.68E-01(+)	3.22E-04(+)	3.84E-04(+)	2.47E-04(+)	2.16E-04(=)	2.46E-01(+)	2.07E-04
	T2	2.29E-04(+)	3.78E-04(+)	3.95E-04(+)	3.45E-04(+)	1.86E-04(=)	2.05E-04(+)	1.78E-04
PIHS	T1	1.22E-03(+)	6.07E-04(+)	4.55E-04(+)	2.89E-02(+)	5.83E-04(+)	1.11E-03(+)	1.89E-04
	T2	6.67E-02(+)	3.43E-02(+)	1.48E-02(+)	2.89E-02(+)	4.44E-02(+)	5.74E-02(+)	3.14E-03
PIMS	T1	4.96E-03(-)	3.21E-03(-)	2.26E-03(-)	5.99E-03(-)	4.19E-03(-)	4.88E-03(-)	1.50E-02
	T2	1.50E+01(+)	1.54E+01(+)	1.36E+01(+)	4.88E+00(+)	1.22E+01(+)	1.73E+01(+)	3.53E+00
PILS	T1	3.26E-04(-)	4.80E-04(+)	4.57E-04(+)	1.49E-03(+)	2.10E-04(-)	2.18E-04(-)	3.87E-04
	T2	6.49E-01(+)	8.76E-03(-)	5.51E-03(-)	4.53E-01(+)	8.14E-03(-)	6.34E-01(+)	6.23E-02
NIHS	T1	6.79E+00(+)	1.53E+00(+)	1.52E+00(+)	1.48E+00(+)	1.53E+00(+)	6.99E+00(+)	1.45E+00
	T2	8.54E-04(+)	4.84E-04(+)	4.93E-04(+)	3.36E-04(+)	3.38E-04(+)	8.24E-04(+)	2.15E-04
NIMS	T1	3.76E-01(+)	1.56E-01(+)	3.76E-01(+)	2.83E-01(+)	1.58E-01(+)	3.02E-01(+)	1.45E-01
	T2	7.59E-02(+)	7.51E-03(+)	1.38E-02(+)	6.49E-03(+)	1.63E-03(+)	5.87E-02(+)	4.08E-04
NILS	T1	8.25E-04(-)	1.18E-03(=)	1.11E-03(=)	1.39E-03(+)	7.84E-04(-)	8.20E-04(-)	1.10E-03
	T2	6.44E-01(+)	6.42E-01(+)	6.42E-01(+)	6.42E-01(+)	6.42E-01(+)	6.42E-01(+)	2.27E-04
		+14/-4/=0	+13/-4/=1	+14/-3/=1	+16/-2/=0	+11/-5/=2	+13/-5/=0	

Table 3
Averaged value of HV obtained by NSGA-II, MOMFEA, MOMFEA-II, MaTDE, MFEA-AKT, MTEA-AD, and CT-EMT-MOES on the classical MOMTO benchmark test suite.

Problem	Task	NSGA-II	MOMFEA	MOMFEA-II	MaTDE	MFEA-AKT	MTEA-AD	CT-EMT-MOES
CIHS	T1	1.44E-01(+)	1.92E-01(+)	1.93E-01(+)	2.10E-01(=)	2.02E-01(=)	1.42E-01(+)	2.10E-01
	T2	1.82E-01(+)	2.29E-01(+)	2.37E-01(+)	3.24E-01(=)	2.66E-01(+)	1.78E-01(+)	3.27E-01
CIMS	T1	8.31E-02(-)	1.81E-01(-)	0.00E+00(=)	0.00E+00(=)	0.00E+00(=)	7.45E-02(-)	0.00E+00
	T2	1.39E-02(-)	1.09E-01(-)	2.04E-01(-)	2.08E-01(-)	2.10E-01(-)	2.08E-01(-)	3.67E-03
CILS	T1	0.00E+00(+)	2.02E-01(+)	2.03E-01(+)	2.08E-01(=)	2.06E-01(+)	0.00E+00(+)	2.10E-01
	T2	6.57E-01(=)	6.53E-01(+)	6.52E-01(+)	6.57E-01(=)	6.60E-01(=)	6.57E-01(=)	6.61E-01
PIHS	T1	6.13E-01(+)	6.34E-01(+)	6.44E-01(+)	0.00E+00(+)	6.39E-01(+)	6.14E-01(+)	6.61E-01
	T2	4.09E-03(+)	5.24E-06(+)	1.59E-01(+)	0.00E+00(+)	1.11E-02(+)	1.69E-02(+)	5.62E-01
PIMS	T1	6.01E-02(-)	1.07E-01(-)	1.31E-01(-)	5.61E-02(-)	8.71E-02(-)	7.43E-02(-)	5.89E-04
	T2	0.00E+00(+)	0.00E+00(+)	0.00E+00(+)	0.00E+00(+)	0.00E+00(+)	0.00E+00(+)	7.33E-03
PILS	T1	2.01E-01(=)	1.95E-01(=)	1.96E-01(=)	1.93E-01(=)	2.06E-01(=)	2.06E-01(=)	2.06E-01
	T2	0.00E+00(=)	1.68E-02(-)	5.74E-02(-)	0.00E+00(=)	2.53E-02(-)	0.00E+00(=)	0.00E+00
NIHS	T1	0.00E+00(=)	0.00E+00(=)	0.00E+00(=)	0.00E+00(=)	0.00E+00(=)	0.00E+00(=)	0.00E+00
	T2	6.26E-01(+)	6.42E-01(+)	6.42E-01(+)	6.58E-01(=)	6.49E-01(+)	6.26E-01(+)	6.61E-01
NIMS	T1	0.00E+00(=)	0.00E+00(=)	0.00E+00(=)	0.00E+00(=)	0.00E+00(=)	0.00E+00(=)	0.00E+00
	T2	1.41E-02(+)	1.09E-01(+)	1.75E-01(+)	1.86E-01(+)	2.86E-01(+)	2.78E-02(+)	3.20E-01
NILS	T1	3.69E-01(-)	3.25E-01(-)	3.27E-01(-)	3.41E-01(-)	3.67E-01(-)	3.73E-01(-)	3.02E-01
	T2	0.00E+00(+)	0.00E+00(+)	0.00E+00(+)	0.00E+00(+)	0.00E+00(+)	0.00E+00(+)	3.26E-01
		+9/-4/=5	+10/-5/=3	+10/-4/=4	+5/-3/=10	+8/-4/=6	+9/-4/=5	

Table 4
Averaged value of EPS_+ obtained by NSGA-II, MOMFEA, MOMFEA-II, MaTDE, MFEA-AKT, MTEA-AD, and CT-EMT-MOES on the classical MOMTO benchmark test suite.

Problem	Task	NSGA-II	MOMFEA	MOMFEA-II	MaTDE	MFEA-AKT	MTEA-AD	CT-EMT-MOES
CIHS	T1	5.37E-02(+)	2.77E-02(+)	2.60E-02(+)	1.26E-02(+)	1.51E-02(+)	5.99E-02(+)	1.10E-02
	T2	2.75E-01(+)	1.64E-01(+)	1.50E-01(+)	4.39E-02(+)	1.04E-01(+)	2.81E-01(+)	1.76E-02
CIMS	T1	3.81E+00(-)	2.69E+00(-)	6.18E+00(+)	5.12E+00(+)	6.64E+00(+)	3.18E+00(-)	4.89E+00
	T2	5.95E-01(+)	1.15E-01(-)	2.07E-02(-)	1.11E-01(-)	1.24E-02(-)	1.24E-02(-)	5.65E-01
CILS	T1	5.59E+00(+)	2.22E-02(+)	2.08E-02(+)	1.57E-02(+)	1.31E-02(+)	5.79E+00(+)	1.27E-02
	T2	1.49E-02(+)	2.29E-02(+)	2.64E-02(+)	2.61E-02(+)	1.27E-02(+)	1.49E-02(+)	1.02E-02
PIHS	T1	3.90E-02(+)	3.46E-02(+)	2.81E-02(+)	1.00E+00(+)	2.81E-02(+)	3.81E-02(+)	1.67E-02
	T2	1.82E+00(+)	1.18E+00(+)	5.22E-01(+)	1.00E+00(+)	1.61E+00(+)	2.04E+00(+)	1.13E-01
PIMS	T1	1.68E-01(-)	1.01E-01(-)	7.51E-02(-)	1.80E-01(-)	1.27E-01(-)	1.46E-01(-)	4.81E-01
	T2	5.64E+02(+)	4.42E+02(+)	4.31E+02(+)	1.55E+02(+)	3.88E+02(+)	5.48E+02(+)	1.03E+02
PILS	T1	1.63E-02(-)	2.66E-02(+)	2.43E-02(+)	4.73E-02(+)	1.27E-02(-)	1.32E-02(-)	1.95E-02
	T2	1.45E+01(+)	2.60E-01(-)	1.67E-01(-)	1.12E+01(+)	2.41E-01(-)	1.45E+01(+)	1.46E+00
NIHS	T1	2.62E+02(+)	3.49E+01(+)	3.47E+01(+)	3.41E+01(+)	3.47E+01(+)	1.59E+02(+)	3.37E+01
	T2	3.02E-02(+)	2.85E-02(+)	3.05E-02(+)	2.48E-02(+)	1.87E-02(+)	3.01E-02(+)	1.22E-02
NIMS	T1	3.98E+01(+)	1.13E+01(+)	2.42E+01(+)	2.86E+01(+)	1.04E+01(+)	1.92E+01(+)	1.02E+01
	T2	3.14E+00(+)	4.34E-01(+)	6.04E-01(+)	4.12E-01(+)	9.19E-02(+)	2.32E+00(+)	2.50E-02
NILS	T1	1.31E-01(-)	1.70E-01(+)	1.80E-01(+)	2.31E-01(+)	1.35E-01(-)	1.38E-01(-)	1.60E-01
	T2	2.10E+01(+)	2.10E+01(+)	2.10E+01(+)	2.10E+01(+)	2.10E+01(+)	2.10E+01(+)	1.70E-02
		+14/-4/=0	+14/-4/=0	+15/-3/=0	+16/-2/=0	+13/-5/=0	+13/-5/=0	

tasking optimization theory is indeed effective. From Table 2, compared to the state-of-the-art EMT algorithms MOMFEA, MOMFEA-II, MaTDE, MFEA-AKT, and MTEA-AD, in terms of the IGD metric, the proposed CT-EMT-MOES obtains superior results on 13, 14, 16, 11, and 13 out of 18 sub-problems respectively in the classical MOMTO test suite. From Table 3, in terms of the HV metric, the proposed CT-EMT-MOES achieves superior performance on 10, 10, 5, 8, and 9 out of 18 sub-problems compared with five state-of-the-art EMT algorithms, respectively. From Table 4, in terms of the EPS_+ metric, in contrast to the five EMT algorithms, the proposed CT-EMT-MOES achieves a better execution on 14, 15, 16, 13, and 13 out of 18 sub-problems, respectively.

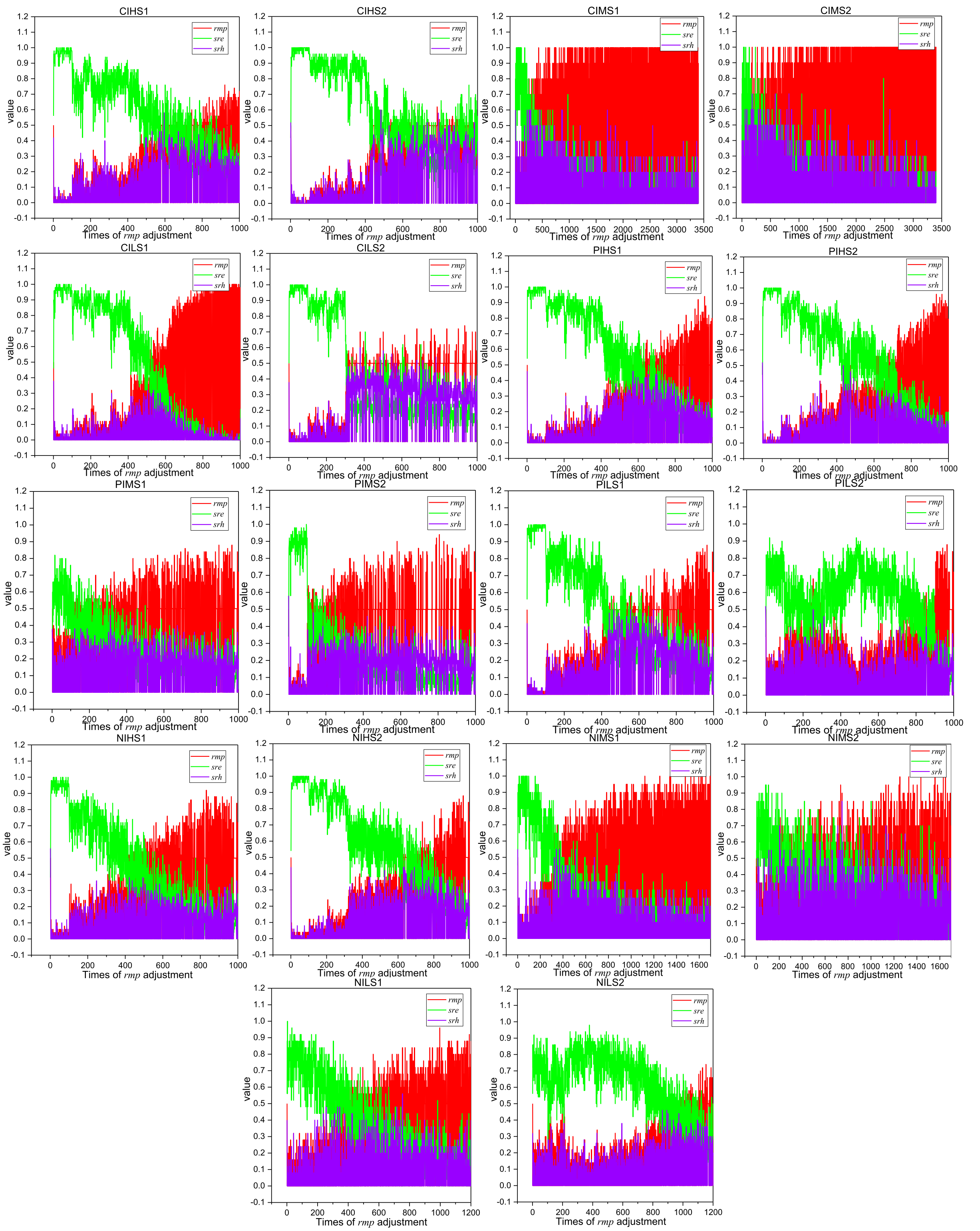

Fig. 10. The values of *rmp*, *sre*, and *srh* in the classical MOMTO benchmark test suites.

The proposed CT-EMT-MOES can achieve the best performance on high-similarity (HS) problems such as CIHS, PIHS, and NIHS. This is because horizontal cultural communication can transfer information from other tasks to the target task. When the two simultaneously optimized tasks possess a high similarity, this strategy can significantly improve the convergence of

Table 5
Averaged value of IGD obtained by NSGA-II, MOMFEA, MOMFEA-II, MaTDE, MFEA-AKT, MTEA-AD, and CT-EMT-MOES on the complex MOMTO benchmark test suite.

Problem	Task	NSGA-II	MOMFEA	MOMFEA-II	MaTDE	MFEA-AKT	MTEA-AD	CT-EMT-MOES
CPLX1	T1	3.85E-04(-)	4.16E-04(-)	4.28E-04(-)	3.12E-04(-)	4.18E-04(-)	4.21E-04(-)	1.14E-03
	T2	6.49E-03(+)	4.51E-03(+)	4.51E-03(+)	5.51E-03(+)	5.50E-03(+)	5.27E-03(+)	7.05E-04
CPLX2	T1	3.98E-04(-)	3.64E-04(-)	3.82E-04(-)	2.90E-04(-)	2.80E-04(-)	4.00E-04(-)	7.71E-04
	T2	1.15E-02(+)	3.73E-03(-)	4.59E-03(-)	1.09E-02(+)	9.04E-04(-)	1.32E-02(+)	6.34E-03
CPLX3	T1	6.04E-03(+)	5.44E-03(+)	4.38E-03(+)	5.67E-03(+)	5.55E-03(+)	5.55E-03(+)	1.05E-03
	T2	2.71E-03(+)	2.96E-03(+)	2.61E-03(+)	4.19E-03(+)	3.15E-03(+)	2.52E-03(+)	8.71E-04
CPLX4	T1	5.72E-03(+)	4.67E-03(+)	4.17E-03(+)	4.94E-03(+)	4.12E-03(+)	5.40E-03(+)	1.10E-03
	T2	7.38E-03(+)	4.62E-03(+)	4.56E-03(+)	6.34E-03(+)	4.79E-03(+)	7.28E-03(+)	1.41E-03
CPLX5	T1	2.43E-03(+)	2.46E-03(+)	2.22E-03(+)	2.93E-03(+)	2.41E-03(+)	3.03E-03(+)	8.29E-04
	T2	6.76E-03(+)	6.57E-03(+)	6.31E-03(+)	2.03E-02(+)	6.49E-03(+)	5.68E-03(+)	2.73E-03
CPLX6	T1	2.90E-03(+)	2.79E-03(+)	2.28E-03(+)	2.94E-03(+)	1.97E-03(+)	2.69E-03(+)	8.29E-04
	T2	7.10E-03(+)	5.76E-03(+)	5.09E-03(+)	6.29E-03(+)	4.51E-03(+)	7.29E-03(+)	1.31E-03
CPLX7	T1	2.96E-03(+)	2.81E-03(+)	2.50E-03(+)	3.40E-03(+)	3.17E-03(+)	2.81E-03(+)	1.33E-03
	T2	2.29E-03(+)	2.62E-03(+)	2.57E-03(+)	4.56E-03(+)	2.78E-03(+)	2.67E-03(+)	6.53E-04
CPLX8	T1	2.36E-03(+)	2.47E-03(+)	2.26E-03(+)	2.00E-03(+)	2.93E-03(+)	2.04E-03(+)	6.38E-04
	T2	1.10E-02(+)	1.49E-02(+)	1.10E-02(+)	1.20E-02(+)	1.39E-02(+)	1.39E-02(+)	9.67E-03
CPLX9	T1	6.52E-03(+)	6.53E-03(+)	6.46E-03(+)	2.56E-02(+)	6.27E-03(+)	6.18E-03(+)	2.89E-03
	T2	6.71E-03(+)	5.67E-03(+)	5.90E-03(+)	5.72E-03(+)	5.00E-03(+)	6.86E-03(+)	9.28E-04
CPLX10	T1	1.07E-02(+)	8.53E-03(+)	9.91E-03(+)	9.77E-03(+)	1.08E-02(+)	9.41E-03(+)	6.95E-03
	T2	1.12E-02(+)	7.02E-03(-)	7.52E-03(-)	1.22E-02(+)	8.49E-03(-)	9.26E-03(-)	9.44E-03
		+18/-2/=0	+16/-4/=0	+16/-4/=0	+18/-2/=0	+16/-4/=0	+17/-3/=0	

all tasks. For problems with no intersection (NI), such as NIHS, NIMS, and NILS-T2, there are competitive relationships between two tasks when simultaneously optimized. The proposed CT-EMT-MOES also achieves the best outcomes for these sub-problems, the reasons for which are summarized as follows: First, the proposed horizontal cultural transmission strategy applies the decision space mapping strategy, which can map the decision space of the source task to that of the target

Table 6
Averaged value of HV obtained by NSGA-II, MOMFEA, MOMFEA-II, MaTDE, MFEA-AKT, MTEA-AD, and CT-EMT-MOES on the complex MOMTO benchmark test suite.

Problem	Task	NSGA-II	MOMFEA	MOMFEA-II	MaTDE	MFEA-AKT	MTEA-AD	CT-EMT-MOES
CPLX1	T1	6.53E-01(-)	6.52E-01(-)	6.52E-01(-)	6.56E-01(-)	6.53E-01(-)	6.52E-01(-)	6.48E-01
	T2	5.47E-01(+)	5.61E-01(+)	5.56E-01(+)	5.24E-01(+)	5.41E-01(+)	5.45E-01(+)	6.51E-01
CPLX2	T1	6.53E-01(=)	6.55E-01(-)	6.54E-01(-)	6.57E-01(-)	6.59E-01(-)	6.53E-01(=)	6.53E-01
	T2	4.86E-01(+)	5.66E-01(-)	5.42E-01(-)	4.46E-01(+)	6.38E-01(-)	4.27E-01(+)	5.13E-01
CPLX3	T1	5.39E-01(+)	5.52E-01(+)	5.61E-01(+)	4.99E-01(+)	5.48E-01(+)	5.37E-01(+)	6.41E-01
	T2	6.31E-01(+)	6.28E-01(+)	6.27E-01(+)	5.87E-01(+)	6.26E-01(+)	6.30E-01(+)	6.45E-01
CPLX4	T1	5.37E-01(+)	5.47E-01(+)	5.62E-01(+)	4.92E-01(+)	5.67E-01(+)	5.40E-01(+)	6.39E-01
	T2	1.94E-01(+)	2.34E-01(+)	2.34E-01(+)	1.89E-01(+)	2.35E-01(+)	1.83E-01(+)	3.03E-01
CPLX5	T1	6.17E-01(+)	6.17E-01(+)	6.19E-01(+)	6.00E-01(+)	6.18E-01(+)	6.20E-01(+)	6.53E-01
	T2	1.92E-01(+)	1.99E-01(+)	2.05E-01(+)	1.37E-01(+)	2.04E-01(+)	2.23E-01(+)	3.70E-01
CPLX6	T1	6.20E-01(+)	6.12E-01(+)	6.18E-01(+)	6.00E-01(+)	6.26E-01(+)	6.17E-01(+)	6.49E-01
	T2	1.74E-01(+)	2.08E-01(+)	2.25E-01(+)	1.93E-01(+)	2.34E-01(+)	1.91E-01(+)	3.02E-01
CPLX7	T1	6.30E-01(+)	6.27E-01(+)	6.27E-01(+)	5.95E-01(+)	6.26E-01(+)	6.30E-01(+)	6.45E-01
	T2	6.26E-01(+)	6.29E-01(+)	6.29E-01(+)	5.85E-01(+)	6.30E-01(+)	6.23E-01(+)	6.52E-01
CPLX8	T1	6.25E-01(+)	6.21E-01(+)	6.22E-01(+)	6.18E-01(+)	6.23E-01(+)	6.23E-01(+)	6.56E-01
	T2	4.75E-01(-)	4.53E-01(-)	4.75E-01(-)	4.36E-01(=)	4.68E-01(-)	4.53E-01(-)	4.39E-01
CPLX9	T1	2.06E-01(+)	2.06E-01(+)	1.91E-01(+)	6.12E-03(+)	2.17E-01(+)	2.04E-01(+)	3.69E-01
	T2	2.21E-01(+)	2.20E-01(+)	2.13E-01(+)	2.03E-01(+)	2.24E-01(+)	1.95E-01(+)	3.12E-01
CPLX10	T1	4.67E-01(+)	5.09E-01(+)	4.79E-01(+)	3.79E-01(+)	4.98E-01(+)	4.77E-01(+)	5.35E-01
	T2	4.38E-01(-)	5.24E-01(-)	5.26E-01(-)	3.74E-01(+)	5.18E-01(-)	4.43E-01(-)	4.33E-01
		+16/-3/=1	+15/-5/=0	+15/-5/=0	+17/-2/=1	+15/-5/=0	+16/-3/=1	

Table 7
Averaged value of EPS_+ obtained by NSGA-II, MOMFEA, MOMFEA-II, MaTDE, MFEA-AKT, MTEA-AD, and CT-EMT-MOES on the complex MOMTO benchmark test suite.

Problem	Task	NSGA-II	MOMFEA	MOMFEA-II	MaTDE	MFEA-AKT	MTEA-AD	CT-EMT-MOES
CPLX1	T1	1.58E-02(-)	1.73E-02(-)	1.87E-02(-)	1.46E-02(-)	1.78E-02(-)	1.82E-02(-)	6.55E-02
	T2	2.13E-01(+)	1.60E-01(+)	1.64E-01(+)	1.96E-01(+)	1.80E-01(+)	1.79E-01(+)	2.99E-02
CPLX2	T1	1.77E-02(-)	1.64E-02(-)	1.71E-02(-)	1.32E-02(-)	1.26E-02(-)	1.76E-02(-)	4.84E-02
	T2	3.53E-01(+)	1.52E-01(-)	1.70E-01(-)	3.27E-01(+)	4.18E-02(-)	4.29E-01(+)	2.98E-01
CPLX3	T1	2.12E-01(+)	1.84E-01(+)	1.57E-01(+)	2.10E-01(+)	1.71E-01(+)	1.87E-01(+)	4.07E-02
	T2	1.45E-01(+)	1.39E-01(+)	1.38E-01(+)	1.89E-01(+)	1.60E-01(+)	1.35E-01(+)	8.54E-02
CPLX4	T1	1.90E-01(+)	1.85E-01(+)	1.64E-01(+)	2.13E-01(+)	1.55E-01(+)	1.77E-01(+)	4.58E-02
	T2	2.06E-01(+)	1.74E-01(+)	1.95E-01(+)	1.94E-01(+)	2.10E-01(+)	2.48E-01(+)	5.92E-02
CPLX5	T1	1.20E-01(+)	1.13E-01(+)	1.07E-01(+)	1.29E-01(+)	1.16E-01(+)	1.29E-01(+)	3.87E-02
	T2	3.33E-01(+)	3.08E-01(+)	2.84E-01(+)	7.70E-01(+)	3.17E-01(+)	2.75E-01(+)	1.57E-01
CPLX6	T1	1.24E-01(+)	1.20E-01(+)	1.11E-01(+)	1.25E-01(+)	8.95E-02(+)	1.22E-01(+)	4.30E-02
	T2	2.11E-01(+)	2.14E-01(+)	2.01E-01(+)	2.07E-01(+)	1.77E-01(+)	2.19E-01(+)	5.93E-02
CPLX7	T1	1.49E-01(+)	1.53E-01(+)	1.47E-01(+)	1.66E-01(+)	1.62E-01(+)	1.39E-01(+)	7.84E-02
	T2	1.08E-01(+)	1.12E-01(+)	1.13E-01(+)	1.93E-01(+)	1.17E-01(+)	1.21E-01(+)	3.53E-02
CPLX8	T1	1.11E-01(+)	1.13E-01(+)	1.13E-01(+)	9.10E-02(+)	1.27E-01(+)	1.05E-01(+)	2.70E-02
	T2	3.44E-01(-)	4.04E-01(+)	3.55E-01(-)	4.06E-01(+)	4.02E-01(+)	3.79E-01(+)	3.75E-01
CPLX9	T1	3.08E-01(+)	3.18E-01(+)	2.94E-01(+)	1.01E+00(+)	2.78E-01(+)	2.90E-01(+)	1.41E-01
	T2	1.92E-01(+)	2.10E-01(+)	1.99E-01(+)	1.82E-01(+)	1.85E-01(+)	2.13E-01(+)	5.69E-02
CPLX10	T1	3.48E-01(+)	2.41E-01(-)	3.18E-01(+)	3.31E-01(+)	3.65E-01(+)	2.97E-01(+)	2.83E-01
	T2	3.31E-01(-)	2.29E-01(-)	2.31E-01(-)	4.23E-01(+)	3.18E-01(-)	3.12E-01(-)	3.35E-01
		+16/-4/=0	+15/-5/=0	+15/-5/=0	+18/-2/=0	+16/-4/=0	+17/-3/=0	

task to reduce the differences between tasks. Second, the proposed adaptive information transfer in the horizontal cultural transmission strategy can dynamically adjust the probability of an information transfer based on the success rate of an individual in the target task. In the right information transfer timing, the probability of an information transfer is adaptively increased, and in the inappropriate timing, the probability will be reduced. Finally, the guided variation strategy can continuously identify potentially better solutions for pushing the population forward. Incidentally, the change curves of *rmp*, *sre*, and *srh* during the iteration in the classical MOMTO benchmark test suite are shown in Fig. 10.

4.6. Performance on complex MOMTO benchmark problems

The IGD, HV, and EPS_+ values of the complex MOMTO benchmark test suites are listed in Table 5, Table 6, and Table 7, respectively. The average metric values over 20 independent runs of each algorithm are demonstrated, and the best result for each sub-problem is marked in gray. In addition, the Wilcoxon rank-sum test at the 95% confidence level was applied to the experimental results for comparing the proposed CT-EMT-MOES with other comparison algorithms. Significantly better, significantly worse, and not comparable results are represented using "+", "−", and "=", respectively. In complex MOMTO benchmark test suite, PSs are more complicated for complex multi-objective problems, and thus traditional MOEA has more difficulty solving such problems. The multi-objective EMT algorithms are better than the conventional single-task multi-objective algorithm NSGA-II on most sub-problems, mainly because the EMT algorithm can introduce more bias to the decision space of the target task through an information transfer.

From Table 5, compared to the state-of-the-art EMT algorithms MOMFEA, MOMFEA-II, MaTDE, MFEA-AKT, and MTEA-AD, in terms of the IGD metric, the proposed CT-EMT-MOES obtains superior results on 16, 16, 18, 16, and 17 out of 20 sub-problems respectively in the complex MOMTO test suite. From Table 6, in terms of the HV metric, the proposed CT-EMT-MOES achieves superior performance on 15, 15, 17, 15, and 16 out of 20 sub-problems compared with five state-of-the-art EMT algorithms, respectively. From Table 7, in terms of the EPS_+ metric, in contrast to the five EMT algorithms, the proposed CT-EMT-MOES achieves a better execution on 15, 15, 18, 16, and 17 out of 20 sub-problems, respectively. This is mainly because the proposed elite-guided variation strategy enables each individual to absorb the PF information of the target task to increase the convergence. The proposed horizontal cultural transmission strategy can introduce various biases into the target task. Because the PS of the complex problem is more complicated, and the PSs of the source and target tasks are quite different, the decision space mapping strategy in the horizontal cultural transmission strategy plays a significant role.

Fig. 11 shows the curves of *rmp*, *sre*, and *srh* in the complex MOMTO benchmark test problems with the iterative process. The initial *rmp* is set to 0.5, to evenly allocate evolutionary resources to the two evolutionary mechanisms of the elite-guided variation strategy and horizontal cultural transmission strategy. After a period of iteration, *sre* begins to decrease, leading to an increase in *rmp*, making the source task transfer more information to the target task.

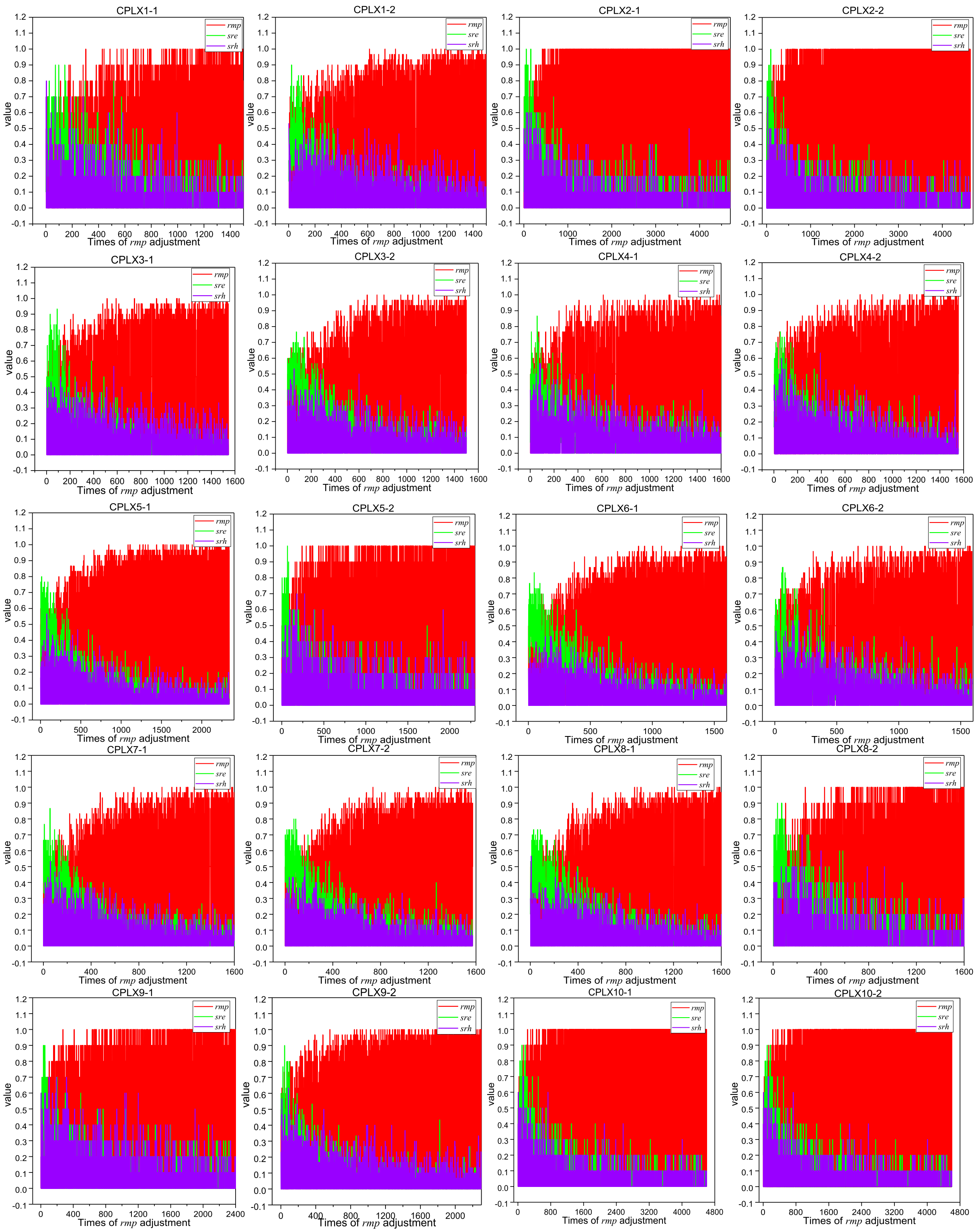

Fig. 11. The values of *rmp*, *sre*, and *srh* in the complex MOMTO benchmark test suites.

Table 8
Averaged value of IGD obtained by EMT-MOES, EMT-MOES-E, EMT-MOES-H, EMT-MOES-EH, and CT-EMT-MOES on the classical MOMTO benchmark test suite.

Problem	Task	EMT-MOES	EMT-MOES-E	EMT-MOES-H	EMT-MOES-EH	CT-EMT-MOES
CIHS	T1	4.14E+00(+)	2.70E-04(+)	3.90E-04(+)	2.15E-04(+)	1.88E-04
	T2	3.66E-01(+)	2.73E-04(+)	5.64E-04(+)	2.00E-04(+)	1.94E-04
CIMS	T1	2.49E-01(+)	1.82E-01(+)	1.38E-01(+)	1.48E-01(+)	1.33E-01
	T2	8.19E-02(+)	1.57E-02(+)	2.05E-02(+)	1.59E-02(+)	1.45E-02
CILS	T1	2.01E-01(+)	2.60E-04(+)	7.62E-04(+)	2.14E-04(+)	2.07E-04
	T2	3.02E-03(+)	2.24E-04(+)	1.84E-04(+)	1.79E-04(=)	1.78E-04
PIHS	T1	4.39E+00(+)	2.91E-04(+)	4.30E-04(+)	1.98E-04(+)	1.89E-04
	T2	1.44E+01(+)	3.33E-03(+)	6.34E-02(+)	3.59E-03(+)	3.14E-03
PIMS	T1	2.83E-02(+)	1.54E-02(+)	1.83E-02(+)	1.67E-02(+)	1.50E-02
	T2	1.03E+01(+)	3.63E+00(+)	6.33E+00(+)	3.96E+00(+)	3.53E+00
PILS	T1	3.24E-02(+)	7.02E-04(+)	1.17E-03(+)	6.22E-04(+)	3.87E-04
	T2	6.24E-01(+)	6.26E-02(+)	5.78E-01(+)	8.93E-02(+)	6.23E-02
NIHS	T1	2.61E+03(+)	1.47E+00(+)	1.50E+00(+)	1.45E+00(=)	1.45E+00
	T2	2.79E+00(+)	2.28E-04(+)	6.04E-04(+)	2.21E-04(+)	2.15E-04
NIMS	T1	6.85E-01(+)	1.53E-01(+)	1.45E-01(=)	1.46E-01(+)	1.45E-01
	T2	1.33E-01(+)	4.94E-04(+)	1.84E-03(+)	6.92E-04(+)	4.08E-04
NILS	T1	1.70E-02(+)	1.15E-03(+)	5.94E-03(+)	1.59E-03(+)	1.10E-03
	T2	3.65E-01(+)	2.58E-04(+)	3.63E-02(+)	2.73E-04(+)	2.27E-04
		+18/-0/=0	+18/-0/=0	+17/-0/=1	+16/-0/=2	

Table 9
Averaged value of HV obtained by EMT-MOES, EMT-MOES-E, EMT-MOES-H, EMT-MOES-EH, and CT-EMT-MOES on the classical MOMTO benchmark test suite.

Problem	Task	EMT-MOES	EMT-MOES-E	EMT-MOES-H	EMT-MOES-EH	CT-EMT-MOES
CIHS	T1	0.00E+00(+)	2.09E-01(=)	2.06E-01(+)	2.10E-01(=)	2.10E-01
	T2	0.00E+00(+)	3.25E-01(=)	3.18E-01(+)	3.26E-01(=)	3.27E-01
CIMS	T1	0.00E+00(=)	0.00E+00(=)	0.00E+00(=)	0.00E+00(=)	0.00E+00
	T2	0.00E+00(+)	3.22E-03(+)	8.24E-04(+)	3.02E-03(+)	3.67E-03
CILS	T1	0.00E+00(+)	2.08E-01(=)	1.97E-01(+)	2.09E-01(=)	2.10E-01
	T2	5.27E-01(+)	6.60E-01(=)	6.61E-01(=)	6.61E-01(=)	6.61E-01
PIHS	T1	0.00E+00(+)	6.58E-01(+)	6.54E-01(+)	6.60E-01(=)	6.61E-01
	T2	0.00E+00(+)	5.24E-01(+)	8.77E-03(+)	5.49E-01(+)	5.62E-01
PIMS	T1	0.00E+00(+)	3.64E-04(+)	0.00E+00(+)	4.24E-04(+)	5.89E-04
	T2	0.00E+00(+)	7.07E-03(+)	0.00E+00(+)	6.84E-03(+)	7.33E-03
PILS	T1	0.00E+00(+)	1.96E-01(+)	1.81E-01(+)	1.99E-01(+)	2.06E-01
	T2	0.00E+00(=)	0.00E+00(=)	0.00E+00(=)	0.00E+00(=)	0.00E+00
NIHS	T1	0.00E+00(=)	0.00E+00(=)	0.00E+00(=)	0.00E+00(=)	0.00E+00
	T2	0.00E+00(+)	6.59E-01(=)	6.54E-01(+)	6.57E-01(+)	6.61E-01
NIMS	T1	0.00E+00(=)	0.00E+00(=)	0.00E+00(=)	0.00E+00(=)	0.00E+00
	T2	0.00E+00(+)	3.16E-01(+)	2.62E-01(+)	3.07E-01(+)	3.20E-01
NILS	T1	0.00E+00(+)	2.81E-01(+)	2.05E-03(+)	1.88E-01(+)	3.02E-01
	T2	0.00E+00(+)	3.25E-01(=)	2.64E-02(+)	3.25E-01(=)	3.26E-01
		+14/-0/=4	+8/-0/=10	+13/-0/=5	+8/-0/=10	

4.7. Discussion of proposed strategies

This section discusses the contribution of each proposed strategy to the performance of the proposed algorithm. Table 8, Table 9, and Table 10 show the IGD, HV, and EPS_+ values of the multi-objective EMT evolution strategy algorithms with different operators on the classical MOMTO benchmark test suite, respectively. The average metric values over 20 independent runs of each algorithm are demonstrated, and the best result for each sub-problem is marked in gray. In addition, the Wilcoxon rank-sum test at the 95% confidence level was applied for the experimental results to compare the proposed CT-EMT-MOES with other comparison algorithms. The significantly better, significantly worse, and not comparable results are represented using "+," "−", and "=", respectively. The EMT-MOES only has the primary mutation operator, EMT-MOES-E only uses the mutation operator and elite-guided variation strategy, EMT-MOES-H only applies the mutation operator and hor-

Table 10
Averaged value of EPS_+ obtained by EMT-MOES, EMT-MOES-E, EMT-MOES-H, EMT-MOES-EH, and CT-EMT-MOES on the classical MOMTO benchmark test suite.

Problem	Task	EMT-MOES	EMT-MOES-E	EMT-MOES-H	EMT-MOES-EH	CT-EMT-MOES
CIHS	T1	1.12E+02(+)	2.10E-02(+)	2.04E-02(+)	1.50E-02(+)	1.10E-02
	T2	1.20E+01(+)	2.87E-02(+)	3.12E-02(+)	2.19E-02(+)	1.76E-02
CIMS	T1	7.95E+00(+)	6.45E+00(+)	5.58E+00(+)	5.54E+00(+)	4.89E+00
	T2	2.44E+00(+)	6.70E-01(+)	6.85E-01(+)	6.13E-01(+)	5.65E-01
CILS	T1	5.69E+00(+)	1.92E-02(+)	4.12E-02(+)	1.66E-02(+)	1.27E-02
	T2	1.18E-01(+)	1.85E-02(+)	1.15E-02(+)	1.37E-02(+)	1.02E-02
PIHS	T1	1.23E+02(+)	1.94E-02(+)	2.76E-02(+)	1.75E-02(+)	1.67E-02
	T2	4.22E+02(+)	1.41E-01(+)	1.55E+00(+)	1.14E-01(=)	1.13E-01
PIMS	T1	8.96E-01(+)	5.05E-01(+)	6.64E-01(+)	5.58E-01(+)	4.81E-01
	T2	3.33E+02(+)	1.13E+02(+)	2.13E+02(+)	1.21E+02(+)	1.03E+02
PILS	T1	8.78E-01(+)	4.15E-02(+)	5.87E-02(+)	3.88E-02(+)	1.95E-02
	T2	1.45E+01(+)	1.73E+00(+)	1.40E+01(+)	2.13E+00(+)	1.46E+00
NIHS	T1	5.06E+04(+)	3.40E+01(+)	3.72E+01(+)	3.39E+01(=)	3.37E+01
	T2	7.08E+01(+)	2.00E-02(+)	2.86E-02(+)	1.59E-02(+)	1.22E-02
NIMS	T1	4.95E+01(+)	1.07E+01(+)	1.04E+01(+)	1.05E+01(+)	1.02E+01
	T2	4.07E+00(+)	4.21E-02(+)	1.34E-01(+)	5.88E-02(+)	2.50E-02
NILS	T1	1.47E+00(+)	1.68E-01(+)	6.11E-01(+)	2.27E-01(+)	1.60E-01
	T2	1.19E+01(+)	2.83E-02(+)	1.15E+00(+)	3.28E-02(+)	1.70E-02
		+18/-0/=0	+18/-0/=0	+18/-0/=0	+16/-0/=2	

izontal cultural transmission strategy, and EMT-MOES-EH not only has the mutation operator and elite-guided variation strategy, but also adopts a horizontal cultural transmission strategy. The *rmp* value was set to 0.5. CT-EMT-MOES is the complete algorithm proposed in this study. It appends the proposed adaptive information-transfer strategy based on EMT-MOES-EH.

Comparing the performances of EMT-MOES, EMT-MOES-E, and EMT-MOES-H, it is evident that both the elite-guided variation strategy and horizontal cultural transmission strategy can significantly improve the performance of the algorithm. Still, the improvement brought about by the elite-guided variation strategy is even greater because the elite-guided variation strategy emphasizes learning from the best individuals and emphasizes aligning with outstanding individuals. This can significantly improve the convergence performance. The horizontal cultural transmission strategy involves learning the relevant knowledge from the source task. Even though the decision space mapping mechanism can appropriately reduce the difference between the source and target tasks, it cannot provide effective information for the target task during the iteration. EMT-MOES-EH is better than EMT-MOES-E and EMT-MOES-H for most tasks, indicating that when the two strategies are used together, the elite information of its own task and the transfer information from the source task can jointly promote a population convergence. However, an equal distribution of evolutionary resources can also lead to the disadvantages discussed in Section 2.3, resulting in a negative transfer. For example, EMT-MOES-EH does not perform better than EMT-MOES-E on CIMS T2, PIHS-T2, PIMS, PILS-T2, NIMS-T2, and NILS. CT-EMT-MOES achieved the best results for all benchmark problems. This is because it adopts the proposed adaptive information transfer strategy based on EMT-MOES-EH, which can adaptively adjust the probability of an information transfer according to the offspring's success rate and reasonably allocate two evolution strategies. The proposed adaptive information transfer strategy is effective for comparing the performances of CT-EMT-MOES and EMT-MOES-EH.

Fig. 12 shows the average IGD numerical curves of EMT-MOES, EMT-MOES-E, EMT-MOES-H, EMT-MOES-EH, and CT-EMT-MOES after running 20 times independently on the classical test suite. Comparing EMT-MOES, EMT-MOES-H, and EMT-MOES-E, it can be observed that both the elite-guided variation strategy and the horizontal cultural transmission strategy can make the algorithm converge faster and that the elite-guided variation strategy can bring about a significant improvement to the performance of the algorithm. Comparing EMT-MOES-E and EMT-MOES-EH, it can be seen that, although EMT-MOES-E can achieve better results than EMT-MOES-EH during the early stage, EMT-MOES-EH can exert a force during the middle and late stages of CIHS, CIMS-T1, CILS, PIHS-T1, PILS-T1, NIHS, and NIMS-T1, and finally achieve better results. This is because most of the individuals in the population are non-dominated in the late stage, and the elite-guided variation strategy is no longer able to bring further improvement to the population. At that moment, the horizontal cultural transmission strategy begins to take effect, and valuable information is transferred from the source task to the target task. However, EMT-MOES-EH performs worse than EMT-MOES-E on CIMS-T2, PIHS-T2, PIMS, PILS-T2, NIMS-T2, and NILS because of the negative transfer caused by not considering the intensity of the information transfer, the timing of information transfer, and the degree of correlation between tasks, as discussed in Section 2.3. The proposed CT-EMT-MOES applies an adaptive

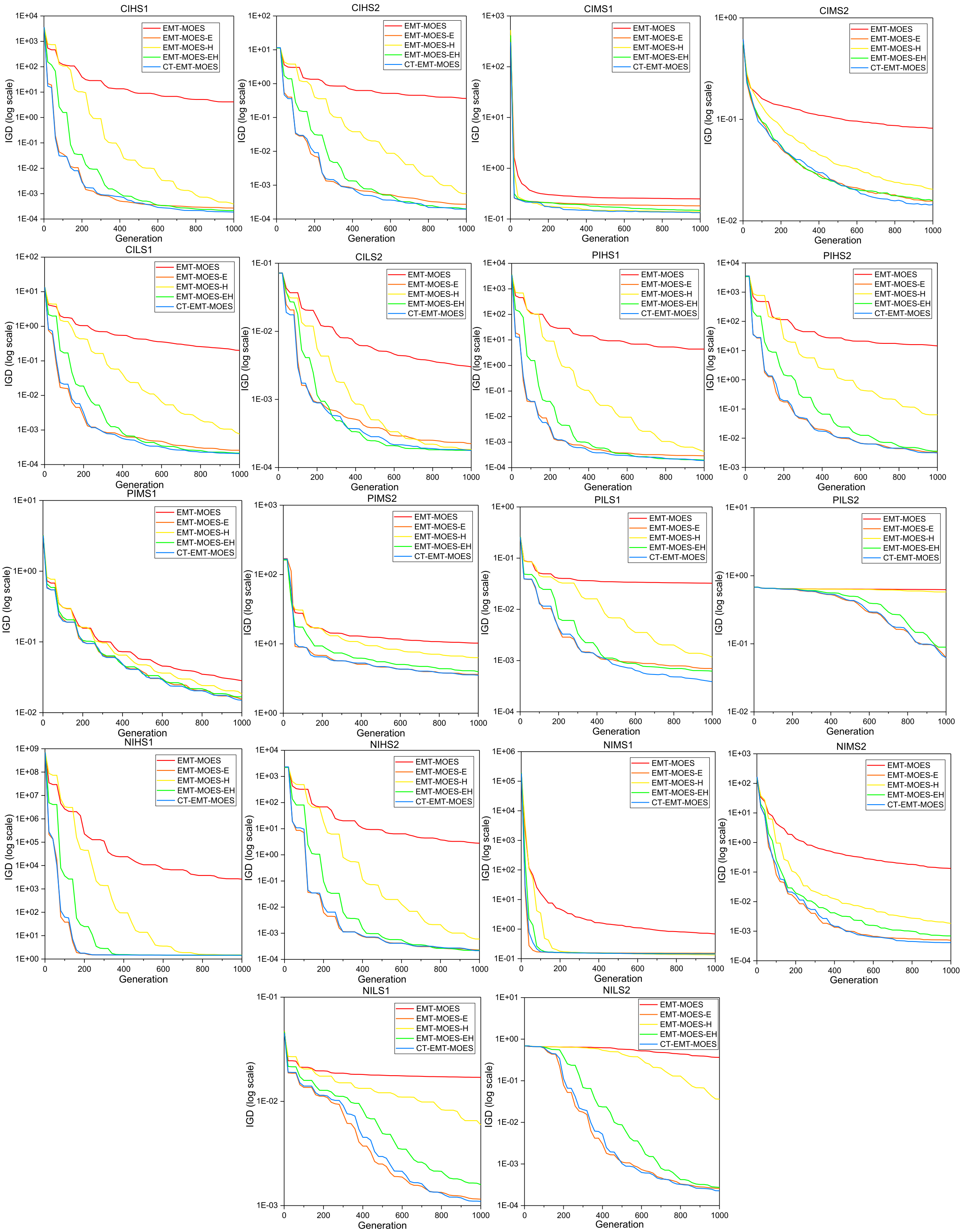

Fig. 12. Curves of IGD values changing with the iteration process in the classical MOMTO benchmark test suite.

information-transfer strategy based on EMT-MOES-EH and can obtain the best results on all sub-problems, proving the effectiveness of the adaptive information transfer strategy.

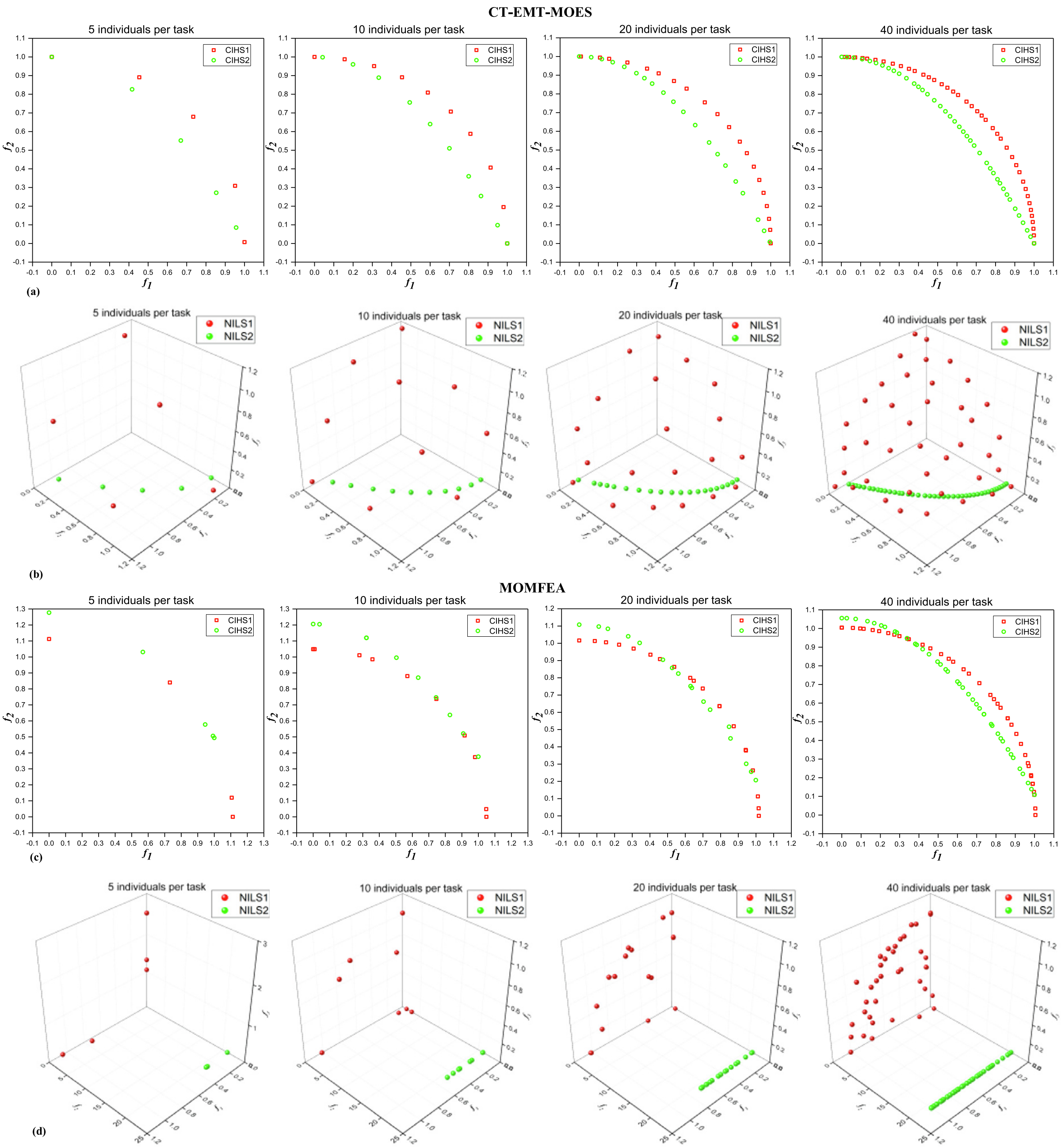

Fig. 13. Experimental results for CIHS and NILS with different population sizes (i.e., 5, 10, 20, and 40) using CT-EMT-MOES and MOMFEA.

4.8. Discussion on the population size

Most multi-objective EMT algorithms need to maintain a large population to expand the search range. When computing resources are limited, the computing platform cannot hold a large population; however, a small population cannot allow the algorithm to converge. The proposed CT-EMT-MOES does not require a large population. The elite-guided variation strategy can refer to individuals for a quick convergence. By contrast, the horizontal and adaptive information transfer strategies can ensure the reliability of an information transfer. The *MED* strategy can automatically lead individuals to hold uniform distances from their nearest neighbors and extend the boundary in the objective space. Fig. 13 shows an objective space used to solve the two simultaneously optimized CIHS and NILS problems using different population sizes, where the evaluation time of each individual is 4000. Even if the population size is small, i.e., only 5, CT-EMT-MOES can still converge well and find the profile of the PF. As the population size gradually increases, the PF obtained is closer to the real PF. By contrast, MOMFEA does not have a fine searching capability in the case of a small population.

5. Conclusion

In this study, a novel multi-objective evolution strategy was proposed for solving multitask optimization problems. Inspired by modern cultural evolution theory, elite-guided variation strategy, and horizontal cultural transmission strategy, two evolutionary operators were proposed. The elite-guided variation strategy can transfer the current PF information to all individuals and quickly converge the population. The horizontal cultural transmission strategy can efficiently share information from the source task to the target task. It can bring about richer diversity to the current population and appropriately promote the convergence of the target task population. To make full use of the two transfer strategies, an adaptive information transfer strategy is proposed to adjust the probability of the information transfer adaptively according to the dominant relationship between the offspring and its parent to reasonably allocate the evolutionary resources. In addition, the evolutionary strategy search engine suffices to guide the population based only on the dominant relationship between the offspring and its parent and can cover the true PF even with a small population size. As the population size increases, the PF obtained becomes more specific. This characteristic makes the evolution strategy extremely efficient when solving subproblems that can only be allocated very few search resources from the total population. Comprehensive experiments were carried out on the classical and complex multi-objective multi-task optimization benchmark test suites to verify the effectiveness of the proposed CT-EMT-MOES. The experimental results demonstrate that the proposed CT-EMT-MOES is superior or comparable to other advanced EMT algorithms.

However, there are still some issues that should be considered in future studies. First, for a multitasking optimization problem, different sub-problems may have various characteristics. With the same population size, the bi-objective subproblem may have an overcrowding 2D PF, whereas the three-objective sub-problem may obtain a very sparse 3D PF. The computation resource allocation strategy still has room for improvement. The proposed CT-EMT-MOES can also be applied to solve expensive MTO, dynamic MTO, large-scale MTO, multimodal MTO, and other more complicated problems. The source-code of CT-EMT-MOES is available publicly at https://github.com/Asurada2015/CT-EMT-MOES.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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