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# Decision variable classification based multi-objective multifactorial memetic algorithm for multi-objective multi-task optimization problem

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## HIGHLIGHTS

- A decision variables classification method based on control variable analysis is proposed.
- An evolutionary recombination strategy based on decision variables' characteristics is proposed.
- The proposed algorithm hybridizes the immune algorithm and evolutionary gradient search as the global and local search operators separately.
- The proposed algorithm can achieve better performance than the state-of-the-art evolutionary multitasking multi-objective evolutionary algorithms.

## ARTICLE INFO

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## ABSTRACT

Multi-task multi-objective optimization problems need to consider the algorithm's convergence and the population's diversity. The information transfer of decision variables with different characteristics may harm the effect of knowledge reuse. This paper proposes a novel hybrid multi-objective multifactorial memetic algorithm to address this issue. The proposed variable classification method will classify decision variables into convergence-related and diversity-related decision variables. Only the same type of decision variables in the source and target tasks can transfer information to avoid negative transfer. Different evolutionary operators are adopted according to the characteristics of decision variables during individual recombination. In addition, the proposed algorithm hybridizes the immune algorithm as the global evolutionary operator and the evolutionary gradient search algorithm as the local search operator into the multifactorial framework to enhance the searching ability. Finally, the proposed algorithm is compared with the state-of-the-art multi-objective evolutionary multitasking algorithms. The results of the experiments show that the proposed algorithm can achieve promising performance on the classical and complex multi-task multi-objective benchmark test suites.

## 1. Introduction

Multi-objective optimization problems (MOPs) require optimizing multiple always conflicting objectives simultaneously, which are ubiquitous in the real world [1–5]. Without loss of generality, assuming that the problem is a minimization problem, it can be defined as:

$$\mathop{\mathrm{Min}}_{\mathbf{x}\in\Omega}(F(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x}), \ldots, f_M(\mathbf{x}))) \tag{1}$$

where $\mathbf{x}$=($x_1$, …, $x_D$) denotes a decision vector with $D$ dimension in the decision space $\Omega$. $F(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x}), \ldots, f_M(\mathbf{x}))$ represents the objective function vector containing the $M$ conflicting objectives functions. Due to the objective functions conflict with each other, optimizing one objective function will inevitably lead to the deterioration of another objective function. It is impossible to identify one single solution optimal for all objective functions. Therefore, Pareto dominance is proposed to determine a set of best tradeoff solutions. Given two decision vector $\mathbf{x}$ and $\mathbf{y}$, if $\forall i \in \{1, 2, 3\ldots, M\}$ $f_i(\mathbf{x}) \leq f_i(\mathbf{y})$ and $\exists j \in \{1, 2, 3\ldots M\}$ $f_j(\mathbf{x}) < f_j(\mathbf{y})$, $\mathbf{x}$ is said to Pareto dominate $\mathbf{y}$ referred to as $\mathbf{x} \prec \mathbf{y}$. If there is no other solution in $\Omega$ that can dominate $\mathbf{x}^*$, then $\mathbf{x}^*$ is called a Pareto optimal solution. All Pareto optimal solutions constitute the Pareto optimal solution set (PS). The objective vectors projected from the PS in the objective space are called the Pareto front (PF).

Multi-objective evolutionary algorithms (MOEAs) have been widely

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used to solve MOPs because they can obtain multiple Pareto optimal solutions in a single run. The traditional MOEAs can be mainly classified into three categories, dominant relationship-based algorithms [6–8], performance indicator-based algorithms [9–11], and decomposition-based algorithms [12–14]. The dominance relationship-based algorithms mainly utilize the dominance relationship as the basis for screening solutions. Improving the traditional dominance relationship is the main thought for these algorithms to increase convergence pressure. The performance indicator-based algorithms usually apply the artificially designed indicator to evaluate the convergence and diversity of the current population to guide it toward the PS. The decomposition-based algorithms generally decompose a complex multi-objective problem into several single-objective problems (SOPs) or multiple simpler MOPs. This type of algorithm purports to simplify the problem, dividing and conquering. All the conventional MOEAs can usually perform decently in traditional MOPs but are designed only to solve one MOP in a single run. When faced with a new problem, the population needs to be reinitialized. However, many MOPs in the real world are interrelated. The knowledge to solve one MOP is beneficial to solve a similar one.

Inspired by the human brains that can parallelly process multiple tasks and flexibly wield the knowledge learned from different tasks, evolutionary multitasking (EMT) is proposed to optimize multiple problems simultaneously [15]. The primary purpose of EMT is to improve the optimization efficiency by transferring and reusing the knowledge learned in optimizing multiple tasks processes. The EMT is very innovative and forward-looking. Once put forward, it has aroused continuous interest and has been successfully applied to solve various real-world optimization problems [16–19]. In the EMT area, one problem that needs to be solved is called a task. The problem that optimizes multiple tasks synchronously is referred to as multi-task problem (MTOP).

When there are MOPs in the simultaneous optimization tasks, it is called a multi-task multi-objective optimization (MOMTO) problem. Unlike single-objective multitasking, multi-objective multitasking aims to find a set of nondominated tradeoff solutions for each MOP rather than a single optimal point. Therefore, the multi-objective EMT algorithms must consider not only the convergence but also the diversity of the population. The non-dominated solutions should be dispersed as much as possible in the search process. In MOPs, different decision variables have otherness characteristics, some decision variables are more related to diversity, and some decision variables are more related to convergence. The diversity-related decision variables promote solutions to evenly dispersed and the convergence-related decision variables boost the population toward the PF. During knowledge transfer, decision variables will also transfer their characteristics to the target task. When decision variables with different characteristics carry out knowledge transfer, it often adversely affects the population.

Existing multi-objective EMT algorithms often neglect this issue, and no research has made a breakthrough. To address this issue, this paper proposes a novel hybrid multi-objective multifactorial memetic algorithm (HMOMFMA). The decision variable classification method is applied to classify the decision variables into convergence-related decision variables and diversity-related decision variables according to their contribution to the convergence and diversity of the population. Specifically, only decision variables of the same category can proceed with knowledge transfer in the knowledge transfer stage. In the individual recombination stage, decision variables of different categories will be recombined using different evolutionary operators. HMOMFMA hybridizes the immune algorithm and evolutionary gradient search (EGS) into the EMT framework to enhance the search ability. The immune algorithm is utilized as the global optimization operator, which has a powerful convergence capacity by focusing on the non-dominated solutions in each iteration and can maintain the population diversity by favoring the sparse solutions selected by the maximum extension distance. The EGS is introduced as the local search operator, which has adaptive mutation length and can guide individual evolution based on gradient information. To verify the efficacy of HMOMFMA, comprehensive empirical studies are conducted on the classical MOMTO and complex MOMTO benchmark problems. The experimental results demonstrate that the proposed HMOMFMA is superior to the state-of-the-art multi-objective EMT algorithms.

The main contribution of this paper are summarized as follow.

1) A decision variables classification method based on control variable analysis is proposed to divide decision variables into two categories according to their contribution to the diversity and convergence of the population.

2) An evolutionary recombination strategy based on decision variables' characteristics is proposed. First, according to their characteristics, two different evolutionary operators are applied to the decision variables in the recombination process. Second, knowledge transfer occurs only between decision variables with the same characteristics.

3) The proposed HMOMFMA hybridizes the immune algorithm and EGS as the global and local search operators separately. The immune algorithm focuses on the sparsest area in the non-dominated solution set that can well maintain population diversity. The EGS performs a local search around the non-dominated solutions and can accelerate convergence by adaptively adjusting the mutation length and guiding the evolution according to the gradient information.

4) To assess the performance of the proposed HMOMFMA, experiments are conducted on the classical and complex benchmark test suites. The proposed HMOMFMA is compared with five state-of-the-art multi-objective EMT algorithms, MOMFEA [35], MOMFEA-II [26], EMT-A [32], MFEA-SADE [36], and MFEA-GHS [37] and a classic MOEA, namely NSGA-II [6]. The experimental results demonstrate that the proposed HMOMFMA is superior to other advanced EMT algorithms.

The rest of this paper is organized as follows. Section 2 reviews the related work of the proposed algorithm. Section 3 describes the details of the proposed HMOMFMA. Section 4 presents the comprehensive experiments to assess the effectiveness of HMOMFMA. Finally, Section 5 concludes this paper and prospects some future directions.

## 2. Related work

### 2.1. Multifactorial algorithm framework

Unlike SOPs and MOPs, MTOPs can be regarded as the third type of optimization problem paradigm: to optimize multiple problems simultaneously and find the optimal solution corresponding to each problem. In MTOPs, each independent problem is called a task, which can be an SOP or a MOP. Assuming that $K$ minimization tasks are optimized simultaneously, multi-task optimization can be defined as Eq. (1).

$$\left\{\mathbf{x}_1^*,\mathbf{x}_2^*,\ldots,\mathbf{x}_K^*\right\} = \{argminT_1(\mathbf{x}_1), argminT_2(\mathbf{x}_2),\ldots, argminT_K(\mathbf{x}_K)\} \tag{2}$$

Where $T_j(j = 1,2,\ldots K)$ denotes the $j$th optimization task, $\mathbf{x}_j$ represents the feasible solutions assigned to the $j$th task, $\mathbf{x}_j^*$ indicates the optimal solution of the $j$th task.

Illuminated by the memetic computing and the multifactorial inheritance [38], Gupta et al. [15] proposed the fundamental EMT algorithm framework based on the memetic principle, namely the multifactorial evolutionary algorithm (MFEA), where each task $T_{j\in\{1,2,\ldots K\}}$ is considered as a memetic factor that influences the individual evolution in the $K$-factorial environment. To achieve efficient cross-domain exchange of genetic material between tasks, MFEA proposed the unified decision space strategy, which encodes each individual's decision variables into a unified space with an equal number of dimensions and uniform upper and lower boundaries to solve the isomerous decision space problem which each task has the diverse number of dimensions and each dimension has different boundaries. Specifically, during population initialization, each individual will be encoded into the unified decision space with $D$ dimensions, where $D = \max\{D_{j\in\{1,2,\ldots,K\}}\}$ is the largest number of dimensions of all the tasks, and each dimension is scaled to between [0,1]. By encoding into the unified decision space, one individual can be regarded as a combination of $K$ chromosomes for different tasks. When solving a specific problem, the individual will be decoded into the corresponding target task space. Suppose $\ddot{\mathbf{x}}_i$ denotes the $i$th individual in the unified decision space, and $\mathbf{x}_i$ represents the result of $\ddot{\mathbf{x}}_i$ after decoding into the target task space. The decoding process is shown in Eq. (2).

$$\mathbf{x}_i = \ddot{\mathbf{x}}_i(1:D_j)\odot(U_j - L_j) + L_j \tag{3}$$

Where $\ddot{x}_i(1:D_j)$ represents the first $D_j$ dimensions of the $i$th individual in the unified decision space, and $D_j$ is the number of dimensions of the $j$th task. $U_j$ and $L_j$ express the upper and lower bounds of the decision variables of the $j$th task, respectively. $\odot$ denotes the Hadamard product.

To evaluate the individuals and compare the performance of individuals in different tasks, for each individual $p_{i\in\{1,2,\ldots,|P|\}}$ in the population $P$, has the following definitions.

**Definition 1**. *(Factorial cost)*: The factorial cost $\psi_j^i$ is designed to measure the performance of indiv$_i$dual $p_i$ on the task $T_j$. When $p_i$ is a feasible solution that satisfies the constraints of $T_j$, $\psi_j^i$ is the value of the objective function of $T_j$. Otherwise, $\psi_j^i$ is a very large real value, which means that $p_i$ is not a feasible solution on task $T_j$ and will be eliminated in the selection process.

**Definition 2**. *(Factorial Rank)*: The factorial rank $r_j^i$ expresses the fitness that $p_i$ can solve the problem $T_j$. Specifically, $r_j^i$ is the index of $p_i$ after the population is arranged in the ascending order of $\psi_j^i$ on task $T_j$.

**Definition 3**. *(Scalar Fitness):* The scalar fitness indicates the best performance that $p_i$ can achieve in all the tasks and is calculated by the best factorial rank, as shown in Eq. (3).

$$\varphi_i = \frac{1}{\min_{j\in\{1,2\ldots k\}} r_j^i} \tag{4}$$

**Definition 4**. *(Skill factor):* The skill factor $\tau_i$ represents the fittest task that $p_i$ can solve, which is the index of the task that $p_i$ achieves the best factorial rank, denoted as Eq. (4).

$$\tau_i = argmin_j\left\{r_j^i\right\} \tag{5}$$

Each individual in the unified decision space can be decoded to be dedicated to a specific task. When the population size is $N$, and the number of tasks is $K$, it is wasteful as the evaluation times will be $N * K$ in one iteration because a solution cannot perform well on all the tasks. Therefore, an individual should ideally be evaluated only on the selected task most likely to perform well. Inspired by memetic computing [38], MFEA proposed the vertical cultural transmission mechanism to address this issue. The main idea is the offspring should share the same memetic environment with the parents by inheriting the task that the parents prefer. The offspring should have the same skill factor as the parent. This mechanism dramatically improves the efficiency of function evaluation. The frequency of function evaluation is reduced by $K$ times compared with the case of evaluating all tasks for each individual.

MFEA provides the basic framework and theories for solving MTOPs, including unified decision space mechanism, solutions encoding and decoding method, and the concept of vertical cultural transmission. These concepts have a profound impact on the EMT algorithms proposed later. The proposed HMOMFMA is also based on the multifactorial framework.

### 2.2. Evolutionary multitasking

Since EMT is an emerging research field, researchers have worked on this from different aspects. To explore and exert the superiority of EMT, various tricks are applied to design and improve EMT algorithms. From the perspective of enhancing recombination strategies, Liu *et al.* [20] present a surrogate-assisted multi-tasking memetic algorithm that the surrogate model with Gaussian process to predict the optimal solution. Feng *et al.* [21] introduce the particle swarm algorithm and differential evolution (DE) algorithm into the EMT field. Song *et al.* [22] extend a dynamic multiswarm algorithm into the EMT area. Each task is arranged in an independent swarm, and then each swarm is partitioned into multiple sub-swarms, using swarm intelligence to optimize.

Considering identifying the most appropriate source task, Zhang *et al.* [23] introduce a probability model learned by the estimated distribution algorithm to represent the distribution of solutions and use the Wasserstein distance to evaluate the similarity between tasks. Chen et al. [24] introduce an archive strategy and a cumulative reward mechanism to measure task similarity by computing the kullback-leibler divergence of archives. Huang *et al.* [25] use the covariance matrix to characterize the distribution of historical solutions for a specific task and select the most suitable task through the similarity of the covariance matrix.

As for adjusting the frequency of knowledge transfer, Bali *et al.* [26] present a data-driven online learning method to optimize the transfer intensity. Zheng *et al.* [27] propose a novel concept named an ability vector to dynamically measure the correlation between the tasks to regulate the transfer intensity in the search process automatically. Li *et al.* [28] propose an adaptive transfer strength strategy, where the knowledge transfer strength is positively correlated with the transfer success rate.

Based on the view of properly allocating search resources, Gong *et al.* [29] propose a dynamic online resource allocation strategy based on problem difficulty, more complex problems will get more computing resources. Yao *et al.* [30] present an EMT algorithm based on decomposition and a dynamic resource allocation strategy, task with faster convergence will be allocated more computing resources. Wen *et al.* [31] propose that computing resources should be reallocated when knowledge transfer starts to fail.

Regarding search space mapping, Feng *et al.* [32] propose a novel task mapping mechanism based on denoising autoencoders (EMT-A), where the solution of the source task can be projected to the target task through the mapping matrix learned by the denoising autoencoder. Bali *et al.* [33] present a linearized domain adaptation method to transform the search space of simple tasks into a reconstruction space highly correlated with complex tasks. Ding *et al.* [34] introduce a novel decision variable transformation strategy to map the solutions of different tasks into a unified space.

As for multi-objective multitasking, the research is still in its infancy, and the literature is not abundant. Gupta *et al.* [35] first introduce the EMT theory into the multi-objective optimization area and propose the multi-objective multifactorial optimization algorithm (MOMFEA). The MOMFEA embeds the classic MOEA NSGA-II into the splendid EMT algorithm framework multifactorial evolutionary algorithm by analogizing the knowledge transfer as the transmission of cultural building blocks in memetic computation to optimize multiple MOPs simultaneously. Liang *et al.* [36] develop a novel multi-objective multifactorial algorithm based on subspace alignment and adaptive differential evolution named MOMFEA-SADE. In MOMFEA-SADE, a mapping matrix obtained by the subspace alignment strategy is introduced to transform the search space to reduce the probability of negative transfer, and an improved adaptive differential evolution is applied as the recombination operator to enhance search efficiency. Liang *et al.* [37] hybridize two novel strategies that genetic transform and hyper-rectangle search into an EMT algorithm, namely MFEA-GHS, which has proved excellent performance in multi-objective multitasking. The genetic

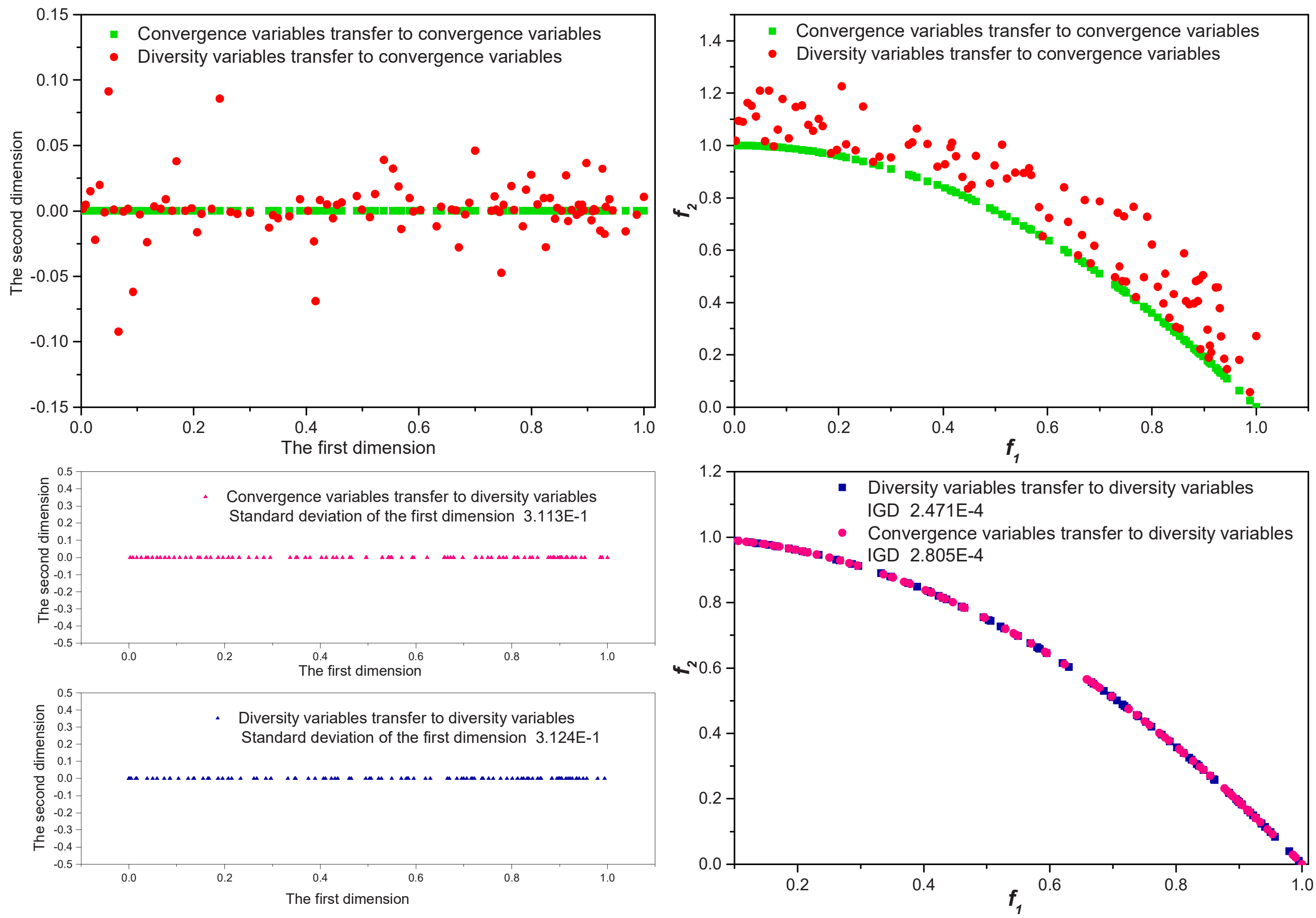


**Fig. 1.** The experiment illustrates the impact of information transfer using different strategies on the target task's decision variables and objective function values. (a) The decision variables values after the information transfer occurs on the convergence-related decision variables. (b) The objective values after the information transfer occurs on the convergence-related decision variables. (c) The decision variables values after the information transfer occurs on the diversity-related decision variables. (d) The objective values after the information transfer occurs on the diversity-related decision variables.

transformation strategy contributes to transforming search space by structuring mapping vectors to improve the efficiency of knowledge transfer. The hyper-rectangle approach constructed based on opposition learning is devoted to expanding the search ability in each subspace.

### 2.3. Motivation

In the MTOPs area, most of the current algorithms treat all the decision variables of the individual in the same way and randomly select the dimensions for information transfer while ignoring the characteristics of different decision variables and their impact on the individual. The decision variables in most MOPs can usually be classified into diversity-related decision variables and convergence-related decision variables. [46] Diversity-related decision variables are committed to the uniform distribution of solutions, and convergence-related decision variables are devoted to converging to the optimal point. In the MTOPs, if information transfer occurs between the convergence-related decision variables in the target task and the diversity-related decision variables in the source task, the distance between the decision variables of this dimension in the target population will increase, which is not conducive to the convergence of the target task. If information transfer occurs between the diversity-related decision variables in the target task and the convergence-related decision variables of the source task, the distance between decision variables of this dimension in the target population will shrink. This does not benefit the uniform distribution of the solution of the target task.

To illustrate the impact of the same and different types of decision variables on the efficiency of EMT during information transfer, an experiment is conducted on the CIHS problem in the classical MOMTO benchmark test suite [40], as shown in Fig. 1. The CIHS-T1 is adopted as the source task, and CIHS-T2 is adopted as the target task. Each task is a bi-objective optimization problem with 50 decision variables. In the algorithm's early and middle stages, the information transfer method is not activated to guarantee fairness. The last-generation population is applied with different transfer strategies.

In CIHS1 and CIHS2, the first decision variables are both diversity-related, while the remaining decision variables are all convergence-related. Fig. 1(a) shows the results of the information transfer of the convergence-related decision variables and diversity-related decision variables in the source task and the convergence-related decision variables of the target task. When the convergence-related decision variables transfer the information to the convergence-related decision variables, the target task still maintains remarkable convergence. But when the diversity-related decision variable in the source task shares information with the convergence-related decision variables in the target task, the convergence-related decision variables of the target task will diffuse to the surroundings. The direct consequence is that the solutions in the objective space will not converge to the true PF. Fig. 1(b) demonstrates the different performances of the solutions in the objective space conducted by the above two transfer strategies. Fig. 1(c) illustrates the results of the information transfer of the convergence-related decision variables and diversity-related decision variables in the source task and the diversity-related decision variables in the target task. It can be seen that after the information transfer of the diversity-related decision variable in the target task and the convergence decision variables in the source task, the distances between individuals in the target task become smaller. The standard deviation of the diversity-related decision variable in the target task is 3.113E-1 after shared information by the convergence-related decision variables, while the standard deviation is 3.124E-1 after shared information by the diversity-related decision variables. Fig. 1(d) demonstrates this point by contrasting the performance of the population in the objective space. The IGD value of solutions after taking in the information from the diversity-related decision variable is 2.471E-4, which is better than the solutions that receive the information from the convergence-related decision variables whose IGD value is 2.805E-4. Therefore, we can conclude that information transfer should be carried out with decision variables of the same type in the source task and the target task, which is beneficial to the effect of EMT. On the contrary, information transfer with decision variables of different types is likely to lead to negative transfer.

Therefore, if the algorithm can first classify the decision variables according to their characteristics related to diversity or convergence and then carry out the corresponding transfer method according to the category of the decision variable, the effect of information transfer can be significantly improved. This paper proposes an improved and efficient way to classify decision variables based on control variable analysis to divide decision variables into diversity and convergence two categories.

## 3. Proposed method

### 3.1. The overall framework of HMOMFMA

The characteristics of different decision variables should be utilized efficiently to improve the efficiency of information transfer. Based on this idea, in the proposed HMOMFMA, the decision variables of each task will be classified as diversity-related decision variables and convergence-related decision variables according to their contribution to the diversity and convergence of the population. Different evolutionary operators will be applied to the decision variable when generating new individuals according to their type. To accelerate the convergence and maintain the diversity, the proposed HMOMFMA hybridizes the immune algorithm [39], focusing on the sparsely distributed non-dominated solutions in each generation. And the proposed HMOMFMA also combines the EGS as the local search operator to improve the search capabilities. The overall framework of the proposed HMOMFMA is summarized in Algorithm 1.

**Algorithm 1.** The overall framework of HMOMFMA.

**Input:** $N$: population size, $N_D$: non-dominated population size, $N_A$: active population size, $N_C$: clone population size, $K$: number of tasks.
**Output:** the final non-dominated solution set.
1. Classify the decision variables for each task.
2. Initialize the population $P$.
3. Assign the skill factor for each individual in $P$ evenly.
4. Evaluate each individual in $P$ according to the skill factor.
5. Select all the non-dominated individuals to form the non-dominated population $P_N$.
6. **While** not reaching the maximum evaluation times **do**
7. Update the active population $P_A$.
8. Generate the clone population $P_C$.
9. Recombine individuals in $P_C$ based on decision variable characteristics
10. Evaluate the offspring and add them into $P_C$'.
11. **If** $u < LS$ **then**, where $u$ is a random number between [0,1].
12. $r$ percent of individuals perform evolutionary gradient search.
13. Evaluate the offspring and add them to $P_C$'.
14. **End if**
15. Assign the skill factor to the individuals in $P_C$' and evaluate.
16. Union $P_N$ and $P_C$', and select all the non-dominated individuals to update $P_N$.
17. **End while**

First, unlike the classic EMT algorithm, HMOMFMA classifies the decision variables according to their contribution to convergence and diversity into two types. This strategy fundamentally determines the evolutionary operator applied to a specific decision variable, the detailed description of the classification method is presented in Section 3.2. Then the population is initialized, and each individual is encoded according to the unified decision space mechanism. All the individuals will be assigned to each task evenly. Each individual will be set the corresponding skill factor and be evaluated in its task. After that, the non-dominated solutions in each task will be picked out to form the non-dominated population represented as $P_N$. In addition to ensuring convergence, diversity is crucial in the iteration process, so more attention should be paid to the sparse areas of the population. Based on this point, $N_A$ solutions with the most sparse distribution in the objective space are selected from $P_N$ to form the active population $P_A$ as the parents of the clone population $P_C$. The sparse degree of the individual is evaluated by the maximum extension distance(MED), which has been proven to be an efficient and helpful method to assess the density of the solution, where a larger MED value indicates the solution is farther away from the other solutions in the objective space. The specific calculation method of MED is shown in Algorithm 2.

The proposed HMOMFEA follows the core idea of the immune algorithm named clonal selection mechanism, which is inspired by the massive asexual reproduction and mitosis of antibody cells in immunology [47]. In the immune system, the genes of the progeny cells are the same as those of the parent cells, which can enhance the binding to the antigen [48]. Clonal selection mechanism believes that better solutions should obtain more clone resources, and the purpose is to generate more local searches around the excellent solution [49]. The allocation of clone resources is carried out according to the MED of the individual in $P_A$, that the greater the MED value, the more clone resources it can gain. The mathematical model for clonal selection is shown in Eq. (5).

$$P_C = \bigcup_{i=1}^{N_A} \{h_i \otimes a_i\}, a_i \in P_A$$

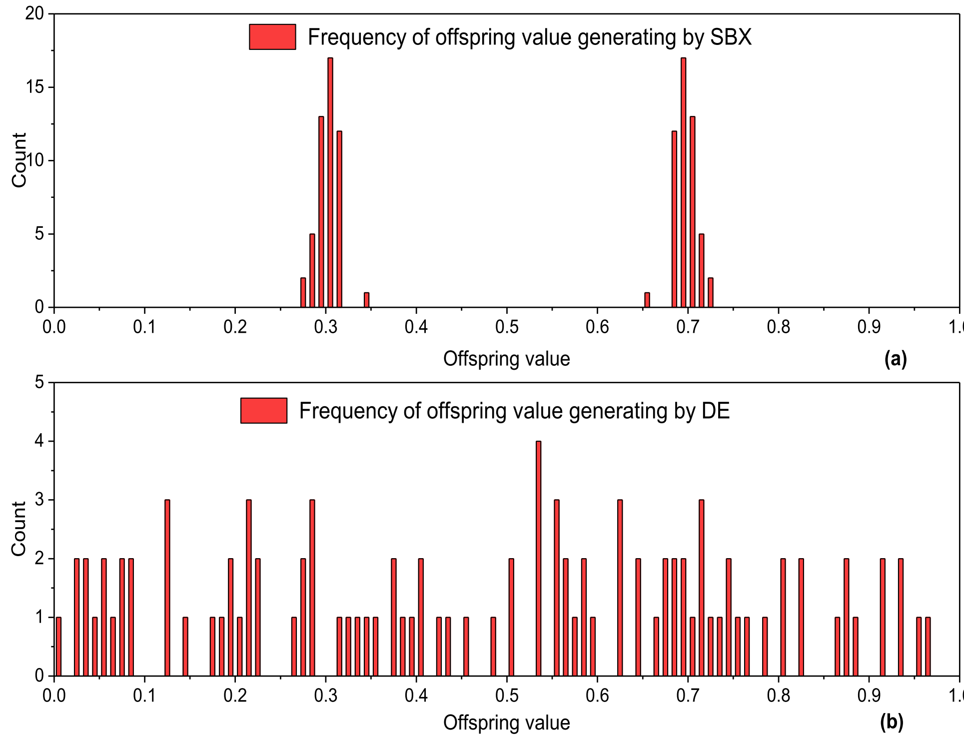


**Fig. 2.** The frequency histograms of the values of 100 offspring generated by SBX and DE respectively.

$$h_i = \left\lceil N_C \times \frac{MED(a_i)}{\sum_{j=1}^{N_A} MED(a_j)} \right\rceil \tag{6}$$

where the operator $\otimes$ indicates the cloning operator and the parameter $h_i$ denotes the number of clones of each solution $a_i$ in $P_A$. Next, the individuals in $P_C$ will undergo recombination and local search to generate the offspring population $P_C{}'$. All the individuals in $P_C$ will be recombined, and the $r$ of the individuals in $P_C$ will perform a local search when the local search threshold is reached. The local search adopts the EGS strategy, as described in Section 3.4. The recombination can be classified into intra-task recombination and inter-task information transfer, and different evolutionary operators are used for recombination according to the types of decision variables, as shown in Section 3.3. Afterward, following the rules of vertical cultural transmission in EMT, each individual in $P_C{}'$ is assigned a skill factor and evaluated on the corresponding task. Finally, union the $P_C{}'$ and $P_N$ in the current generation and pick out all the non-dominated solutions to form the new $P_N$ of the next generation.

**Algorithm 2.** The maximum extension distance.

**Input:** *idx*: the index of the current individual in the population, *N*: the population size, *M*: the number of objectives of the current task.
**Output:** the *MED* of the current individual.
Set *TotalDis*=0.
Set *NearDis*=+∞.
**For** *i*=1 to *N*
    **If** (*i*==*idx*) **then** continue.
    **End if**
    Set *Dis*=0.
    **For** *j*=1 to *M*
        $Dis=Dis+|f_j^{(i)} - f_j^{(idx)}|$.
    **End for**
    *TotalDis*=*TotalDis*+*Dis*.
    **If** (*Dis*<*NearDis*) **then** *NearDis* = *Dis*.
    **End if**
**End for**
*MED* = *NearDis*×*TotalDis*.

### 3.2. The decision variable classification method for information transfer

The specific pseudo code of proposed decision variable classification method is shown in Algorithm 3.

**Algorithm 3**. The proposed decision variable classification method.

**Input:** $D$: the number of the decision variables in a specific task, $NS$: the number of sampling points from a single dimension of decision variables.
**Output:** $IC$: the set of indexes of the convergence-related decision variables, $ID$: the set of indexes of the diversity-related decision variables.
1. $IC=\emptyset$, $ID=\emptyset$, $S=\emptyset$
2. **For** $i$=1 to $D$
3. $\quad x_i=(u_i+l_i)/2$.
4. **End for**
5. **For** $i$=1 to $D$
6. $\quad$ **For** $j$=1 to $NS$
7. $\quad\quad \mathbf{x'}=\mathbf{x}$.
8. $\quad\quad x'_i=l_i+\frac{j}{N_s}*(u_i-l_i)$.
9. $\quad\quad$ add $\mathbf{x'}$ to the archive $S$ as $\mathbf{s_j}$ and set $n_j$=0.
10. $\quad$ **End for**
11. $\quad$ Evaluate all the solutions in $S$, and set $NF$=0.
12. $\quad$ **For** $j$=1 to $NS$
13. $\quad\quad$ **For** $k$=1 *to* $NS$
14. $\quad\quad\quad$ **If** ($\mathbf{s_k}\prec\mathbf{s_j}$) **then**
15. $\quad\quad\quad\quad n_j=n_j+1$.
16. $\quad\quad\quad$ **End if**
17. $\quad\quad$ **End for**
18. $\quad\quad$ **If** ($n_j$ == 0) **then**
19. $\quad\quad\quad NF=NF+1$.
20. $\quad\quad$ **End if**
21. $\quad$ **End for**
22. $\quad$ **If** ($NF$ == 1) **then**
23. $\quad\quad$ add index $i$ into $IC$.
24. $\quad$ **Else**
25. $\quad\quad$ add index $i$ into $ID$.
26. $\quad$ **End if**
27. **End for**

Firstly, a template solution for control variable analysis is initialized. All its dimensions are set to the mean value of the upper and lower bounds of the corresponding dimensions of the specific task, as shown in line 3. $u_i$ and $l_i$ represent the upper bound and the lower bound of the $i$th decision variable in this task respectively. Then, based on the template solution, $NS$ solutions with different values of the $i$th variable but other dimensions remaining unchanged are generated to perform control variable analysis on the $i$th dimension, as shown in line 8, and these solutions are saved into the archive $S$. Next, the solutions in archive $S$ are compared based on the dominance relationship. Suppose $n_j$, which represents the number of individuals dominating the $j$th individual in the archive, is zero. In that case, that means there not exists a solution that can dominate $\mathbf{s_i}$ in the archive $S$, and $\mathbf{s_i}$ is on the non-dominated front. And once such a non-dominated solution is found, $NF$ plus one, where $NF$ represents the total number of non-dominated solutions in archive $S$. Finally, if $NF$ is equal to one, the $i$th dimension decision variable will be convergence-related, and the index $i$ is stored in the $IC$ set. Otherwise, the $i$th dimension decision variable is regarded as a diversity-related decision variable. The index $i$ is stored in the $ID$ set.

### 3.3. Recombination based on the decision variable characteristic

Effectively utilizing the characteristics of different decision variables can speed up the algorithm convergence and ensure the diversity of the population. The proposed HMOMFMA applies different recombination operators to the diversity-related and convergence-related decision variables. Regarding diversity-related decision variables, the offspring should be far away from the parent. For convergence-related decision variables, the offspring should be near the parent and performs a local search around the parent. Therefore, the differential evolutionary operator is the recombination operator for diversity decision variables. The DE operator will utilize the information of two additional solutions randomly selected from the population to optimize the current solution, and the generated offspring are less similar to the parent. For convergence decision variables, the SBX operator is applied to the recombination operator, and the generated offspring is near the parent, which can exploit more promising solutions in the local range while ensuring the effective convergence of the algorithm. Fig. 2 shows an example of the histogram of the offspring generated by the two operators respectively, where the values of the parent $\mathbf{x_1}$ and $\mathbf{x_2}$ are 0.3 and 0.7, and the value of the third parent $\mathbf{x_3}$ in the DE operator is randomly selected from the decision space. The parameter $\eta$ in SBX is set to 20, the parameter $F$ in DE is set to 1, and both operators are tested 100 times respectively.

If the knowledge of other tasks can be efficiently utilized in the offspring generation process, the efficiency of the optimizing target task can be significantly improved. This is the main advantage and essential feature of the EMT algorithm. Thus, in the proposed HMOMFMA, the genetic mapping transfer strategy [37] is used to map individuals far apart in the unified decision space to the areas close to the target task. The genetic mapping transfer strategy can significantly improve the information transfer efficiency of the source task and reduce the negative transfer. The mathematical expression is shown in Eq. (6).

$$x_i^{target} = x_j^{source} \times \frac{\overline{x_i^{target}} + \breve{\varepsilon}}{\overline{x_j^{source}} + \breve{\varepsilon}}, i \in \{1, \ldots, D_{targer}\}, j \in \{1, \ldots, D_{source}\} \tag{7}$$

$x_i^{target}$ represents the transfer dimension in the target task, and $\overline{x_i^{target}}$

**Table 1**
Parameters setting for MOMTO experiments.

| Parameter | Description | HMOMFMA | MOMFEA | MOMFEA-II | EMT-A | MFEA-SADE | MFEA-GHS | NSGA-II |
|---|---|---|---|---|---|---|---|---|
| *rmp* | Random mating probability | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | - |
| *N* | Population size for all the task | 200 | 200 | 200 | 200 | 200 | 200 | 200 |
| *max FEs* | Maximum number of evaluations | 200,000 | 200,000 | 200,000 | 200,000 | 200,000 | 200,000 | 200,000 |
| $p_c$ | Crossover probability | 0.9 | 0.9 | 0.9 | 0.9 | | 0.9 | 0.9 |
| $\eta_c$ | Distribution index of crossover | 20 | 20 | 20 | 20 | | 20 | 20 |
| $p_m$ | Mutation probability | $1/D$ | $1/D$ | $1/D$ | $1/D$ | | $1/D$ | $1/D$ |
| $\eta_m$ | Distribution index of mutation | 20 | 20 | 20 | 20 | | 20 | 20 |
| $N_A$ | Population size of active population | 100 | - | | | | - | - |
| $N_C$ | Population size of clone population | 200 | - | | | | - | - |
| *F* | Scaling factor in DE | 0.5 | - | | | | - | - |
| $\varepsilon$ | Step size factor in EGS | 1.8 | - | | | | - | - |
| *LS* | Local search rate in EGS | 0.02 | - | | | | - | - |
| *r* | Local search proportion in EGS | 0.05 | - | | | | - | - |
| *LST* | Local search times in EGS | 4 | - | | | | - | - |
| *L* | Number of trial solution in EGS | 5 | - | | | | - | - |

expresses the mean value of this dimension. $x_j^{source}$ denotes the transfer dimension in the source task, $\overline{x_j^{source}}$ is the mean value of this dimension, and $\breve{\varepsilon}$ indicates a tiny real value. Note that the transfer dimensions of the target task and the source task are not one-to-one correspondence but are selected according to the characteristics of the decision variables. When recombination proceedings, if the skill factors of parents are detected to be inconsistent, the genetic mapping transfer strategy will be activated to carry out the information transfer. One point that needs special attention is that since convergence-related decision variables require a local search in the vicinity of the parent, receiving all information from other tasks indiscriminately is inappropriate. The proposed HMOMFMA inherits the random mating probability (*rmp*) concept in MOMFEA. When *rmp* is met, cross-task knowledge transfer on convergence-related decision variables can be performed. If the skill factors of the parents are the same, the recombination is performed directly without activating the genetic mapping transfer strategy. After completing the recombination, if the mutation condition is met, the polynomial mutation mutates the decision variable. The pseudo-code of the recombination method based on the decision variable characteristic is shown in Algorithm 4.

**Algorithm 4.** The recombination method based on the decision variable characteristic.

**Input:** **a**: parent individual from the $P_C$, $IC$: the set of indexes of the convergence-related decision variables, $ID$: the set of indexes of the diversity-related decision variables.
**Output:** **o**: the generated offspring.
Randomly select two individuals **b** and **c** from the $P_N$.
**For** $i$=1 to $D_{\mathbf{a}}$,
    **If** $i$ in $ID$
        Randomly select two diversity-related decision variables $b_{div}$ and $c_{div}$.
        **If** ($\tau_{\mathbf{b}}$ != $\tau_{\mathbf{a}}$) **then**
            Perform genetic mapping transfer strategy to $b_{div}$.
        **End if**
        **If** ($\tau_{\mathbf{c}}$ != $\tau_{\mathbf{a}}$) **then**
            Perform genetic mapping transfer strategy to $c_{div}$.
        **End if**
        $o_i = DE\,(a_i, b_{div}, c_{div})$.
    **End if**
    **If** $i$ in $IC$
        Randomly select a convergence-related decision variable $b_{cov}$ from **b**.
        **If** ($\tau_{\mathbf{b}}$ != $\tau_{\mathbf{a}}$) and $rand1 < rmp$ **then**,
            Perform genetic mapping transfer strategy to $b_{cov}$.
        **Else** $o_i = polynomial\ mutation\ (a_i)$.
        **End if**
        $o_i = SBX\,(a_i, b_{cov})$.
    **End if**
    **If** $rand2 < p_m$ **then**,
        $o_i = polynomial\ mutation\ (o_i)$
    **End if**
**End for**

### 3.4. Local search based on evolutionary gradient search

To effectively search for better solutions around high-quality solutions, which means the non-dominated solutions with sparse distribution, HMOMFMA uses a combination of global search and local search. The immune algorithm is applied as a global search optimizer, and EGS is used as a local search optimizer. The essential idea of the EGS is to follow the gradient information obtained in the evolution process to guide the population to move to optimal solutions. In SOPs, the deviation of the objective function value is usually used as the gradient information. In MOPs, combining the characteristics that the population commits to pressing close to the PF and evenly distributed, the proposed HMOMFMA applies the average value of the normalized objective function value as the fitness $F(x)$ to evaluate the performance of the individual to obtain the gradient information in the iterative process. The specific pseudo code of EGS in HMOMFMA is shown in Algorithm 5.

**Algorithm 5.** Local search based on evolutionary gradient search.

**Input:** $\mathbf{a}$: the parent individual from the $P_C$, $L$: the number of trial solutions, *LST:* the total iteration times performing EGS.
**Output:** $\breve{\mathbf{a}}$: the final individual after local search.

1. $\breve{\mathbf{a}}=\mathbf{a}$.
2. **For** $t$=1 to *LST*
3.     **For** $i$=1 to $L$
4.         Create trial solution $\mathbf{r}_i$ by perturbing $\breve{\mathbf{a}}$ using mutation $N\,(0,{\sigma_t}^2)$.
5.         Set skill factor and evaluate $\mathbf{r}_i$.
6.     **End for**
7.     Assign fitness $F(\mathbf{r}_i)$ to each individual $\mathbf{r}_i$.
8.     $\hat{v}=\frac{\sum_{i=1}^{L}[F(\mathbf{r}_i)-F(\breve{\mathbf{a}})]\cdot(\mathbf{r}_i-\breve{\mathbf{a}})}{\|\sum_{i=1}^{L}[F(\mathbf{r}_i)-F(\breve{\mathbf{a}})]\cdot(\mathbf{r}_i-\breve{\mathbf{a}})\|}$, where $v$ is the estimated global gradient direction.
9.     $\mathbf{o}=\breve{\mathbf{a}}-\sigma_t\hat{v}$.
10.     $\sigma_{t+1}=\begin{cases}\sigma_t\varepsilon & if\ \mathbf{o}\prec\breve{\mathbf{a}}\\ \sigma_{t/\varepsilon} & if\ \breve{\mathbf{a}}\prec\mathbf{o}\end{cases}$.
11.     **If** $\mathbf{o}\prec\breve{\mathbf{a}}$ **then** $\breve{\mathbf{a}}=\mathbf{o}$.
12.     **End if**
13. **End for**

EGS mainly consists of two steps, estimating the gradient direction through evolution and updating the solution using gradient descent. To estimate the gradient direction, first, the $L$ trial solutions $\mathbf{r}_i$ are generated by perturbing the parent by the normal distribution $N\,(0,{\sigma_t}^2)$, where $\sigma_t$ controls the mutation strength. Then, according to the principle of vertical cultural transmission, the skill factor is assigned to $\mathbf{r}_i$ and evaluated. The calculation of the gradient requires one single fitness index. Here the average of the normalized objective value is utilized as the fitness value. Afterward, the gradient is calculated according to the method shown in line 8. Next, the offspring solution $\mathbf{o}$ is generated using the gradient descent method, as shown in line 9. Then, according to the dominant relationship between the parent individual $\breve{\mathbf{a}}$ and the offspring individual $\mathbf{o}$, the mutation step $\sigma_t$ is updated. If the offspring $\mathbf{o}$ can dominate parent $\breve{\mathbf{a}}$, then the current $\breve{\mathbf{a}}$ will be replaced by the offspring $\mathbf{o}$, and the $\sigma_t$ will be multiplied by the coefficient $\varepsilon$. Otherwise, the $\sigma_t$ will be divided by the coefficient $\varepsilon$, where the coefficient $\varepsilon$ is generally set to 1.8 [41]. Finally, when the local search times reach *LST*, the local search process is terminated, and the final individual $\breve{\mathbf{a}}$ is output as the result of the local search.

### 3.5. Complexity analysis

In this section, the computational complexity of one generation of the proposed HMOMFMA is discussed. Suppose $d$ denotes the total number of the dimensions of the unified decision space, $N_D$ indicates the population size of the nondominated population, $N_C$ expresses the population size of the clone population, $N_A$ expresses the population size of the active population, $m$ indicates the total number of the objective functions, $NS$ is the number of sampling points from a single dimension of decision variables. Before the population is initialized, the decision variables will be classified. This classification is one-time, so the algorithm complexity is $O(dNS^2)$. In the environment selection, the time complexities of the nondominated sorting is $O(m(N_D+N_C)^2)$, the calculating of the MED is $O(mN_A)$, and the updating of the archive active population is $O(mN_D logN_D)$, respectively. The time complexity of the information transfer is $O(dN_D)$. The time complexity of the cloning is $O(N_C)$, and the time complexity of the recombination and hypermutation is $O(dN_C)$. For the local search operator EGS, its time complexity is related to the depth of its search and the probability of occurrence. Its time complexity is $O(LS\times LST\times r\times L\times N_D)$. Overall, the total computational complexity of HMOMFMA is $(O(m(N_D+N_C)^2)+O(MN_A)+O(mN_D logN_D)+O(dN_D)+O(N_C)+O(dN_C))\text{x}O(LS\times LST\times r\times L\times N_D)$. According to the operation rules of symbol $O$, the time complexity of the proposed HMOMFMA can be simplified as $O(m\times(N_D+N_C)^2\times LS\times LST\times r\times L\times N_D)$. However, considering that the number and probability of local search usage is very small, in actual use, the speed of the algorithm will not be significantly reduced.

## 4. Experiments

### 4.1. Test suites introduction

In MTOPs, the similarity of the fitness landscape and the degree of intersection of optimal solutions are the two most important factors affecting the effectiveness of genetic information transfer between tasks. If the values of the corresponding dimensions of the optimal solutions of different tasks are closer, the transfer of genetic information between tasks is more conducive to optimization. Likewise, the more similar the fitness landscapes of optimization functions for different tasks are, the more knowledge individuals learn from source tasks can indirectly help optimize target tasks. According to the intersection degree of the global optimum, the classic MOMTO benchmark test problems are designed into three categories: complete intersection (CI), partial intersection (PI), and non-intersection (NI). According to the similarity of the fitness landscape, the classic MOMTO benchmark test problems can be divided into three categories of high similarity (HS), medium similarity (MS), and low similarity (LS). The classic MOMTO benchmark suite consists of nine consecutive multi-objective problems based on combining the above two classification strategies. Details of the classic MOMTO benchmark test suite can be found in [40]. The complex MOMTO benchmark test suite named CPLX is first introduced in the IEEE CEC 2019 competition on evolutionary multi-task optimization [42], which is more difficult than the classical test suite. The sub-problem of it is designed according to [43].

### 4.2. Compared algorithms

The proposed HMOMFMA will be compared with five state-of-the-art

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multi-objective EMT algorithms MOMFEA [35], MOMFEA-II [26], EMT-A [32], MFEA-SADE [36], and MFEA-GHS [37] and a classic MOEA, namely NSGA-II [6]. MOMFEA [35] is the earliest and most classic multi-objective EMT algorithm, which can be regarded as the benchmark of multi-objective EMT algorithm. The well-known NSGA-II [6] is the basis for MOMFEA to apply the multitasking theory to process MTOPs. By comparing the performance of MOMFEA and NSGA-II, it can show the advantages of the multi-objective EMT algorithm over traditional MOEA, so NSGA-II is also included as one of the comparison algorithms. MOMFEA-II [26] applied a data-driven online learning method to optimize the transfer intensity during the search process to solve MOMTO problems and reduce negative transfers. EMT-A [32] applies the denoising autoencoder in the MFEA to map different tasks' decision spaces. MFEA-SADE [36] employs the subspace alignment strategy and adaptive differential evolutionary operator into MFEA. MFEA-GHS [37] is the improved version of MFEA that introduces genetic transform and hyper-rectangle search strategies into MFEA. All the algorithms are implemented utilizing the Jmetal 4.5.2 [45]. The platform on which the algorithm runs is a PC with Intel Core i5–9400 F CPU 2.90 GHz, and 16.00 GB of RAM.

### 4.3. Parameter settings

For a fair comparison, in MOMFEA, MOMFEA-II, EMT-A, MFEA-SADE, and MFEA-GHS, the population size is set to 200, but in NSGA-II, the population size for each task is set to 100. For the EMT algorithms, the maximum number of fitness evaluations is 200,000, but for the conventional MOEA like NSGA-II, the maximum number of evaluations for each task is set to 100,000. The parameter settings of the comparison algorithms are consistent with the original paper. The details of all the parameter settings are summarized in Table 1.

### 4.4. Performance indicators

For each benchmark problem, 10,000 points in the three-objective task and 1000 points in the two-objective task are sampling in true PF to evaluate the inverted generational distance (IGD) [44]. The IGD indicator measures the distance between the true PF and the closest individual in the obtained solutions. The indicator can be expressed as Eq. 7. where $Dist_i$ is the Euclidean Distance between the $ith$ solution in the true PF and the closest individual in the obtained solutions and $N_{PF}$ is the number of the sampling points. The lower the IGD value is, the better the convergence and diversity of the population.

$$\mathrm{IGD} = \frac{\left(\sum_{i=1}^{N_{PF}} Dist_i^2\right)^{1/2}}{N_{PF}} \tag{8}$$

### 4.5. Performance on classical MOMTO benchmark test suite

The experimental result of the classical MOMTO benchmark test suite is shown in Table 2. The average and standard deviation of IGD values over 20 independent runs of each algorithm are demonstrated, and the best result on each sub-problem is marked in gray. The Wilcoxon rank-sum test at the 95% confidence level was applied for the experimental results to compare the proposed HMOMFMA with other comparison algorithms. The significantly better, significantly worse, and not comparable results are represented using "+ ," "-," and "= ," respectively. In the classical MOMTO benchmark test suites, the multi-objective EMT algorithm can defeat the classic single-task MOEA NSGA-II on most benchmark test problems. This is mainly due to the knowledge-sharing and transferring mechanism of the EMT algorithm, which indicates that the multitasking optimization theory is indeed effective. From Table 2, compared to the state-of-the-art MOEAs MOMFEA, MOMFEA-II, EMT-A, MFEA-SADE, MFEA-GHS and NSGA-II, in terms of the IGD metric, the proposed HMOMFMA obtains superior results on 15, 15, 13, 15, 16 and 16 out of 20 sub-problems respectively in the complex MOMTO test suite.

Concerning the classical MOMTO benchmark test suite, the proposed HMOMFMA can perform best on high similarity (HS) problems such as CIHS-T1, PIHS, NIHS. This is because HMOMFMA classifies the decision variables according to their contribution to the diversity and convergence of the population and only transfers knowledge between decision variables of the same type between tasks, which makes the algorithm not interfere with the phased results of the target task and can effectively improve the knowledge reuse rate. When the two simultaneously optimized tasks possess high similarity, this strategy can significantly enhance algorithm performance. For problems with low similarities, such as PILS and NILS-T2, even if the coincidence degree of the optimum points is not high, the local search strategy based on EGS can constantly dig out potential better solutions to push the population forward. Therefore, HMOMFMA has also achieved significant advantages in these problems.

### 4.6. Performance on complex MOMTO benchmark test suite

The experimental results of the complex MOMTO benchmark test suite are shown in Table 3. The average and standard deviation of IGD

**Table 2**
Averaged value and standard deviation of the IGD on the classical MOMTO test suite.

| Problem | Task | HMOMFMA | MOMFEA | MOMFEA-II | EMT-A | MFEA-SADE | MFEA-GHS | NSGA-II |
|---|---|---|---|---|---|---|---|---|
| CIHS | T1 | 3.35E-04 | 3.80E-04(+) | 4.69E-04(+) | 3.65E-04(+) | 3.89E-03(+) | 1.57E-03(+) | 1.97E-03(+) |
| | T2 | 8.64E-04 | 2.66E-03(+) | 2.36E-03(+) | 2.15E-04(-) | 9.50E-04(+) | 5.69E-03(+) | 4.36E-03(+) |
| CIMS | T1 | 4.19E-02 | 6.62E-02(+) | 1.76E-01(+) | 1.82E-01(+) | 3.26E-04(-) | 1.09E-01(+) | 1.14E-01(+) |
| | T2 | 5.72E-03 | 1.13E-02(+) | 3.88E-04(-) | 2.01E-02(+) | 8.25E-04(-) | 1.42E-02(+) | 2.34E-02(+) |
| CILS | T1 | 8.30E-02 | 3.16E-04(-) | 3.84E-04(-) | 2.46E-04(-) | 3.91E-03(-) | 6.04E-01(+) | 2.84E-01(+) |
| | T2 | 3.28E-04 | 1.98E-04(-) | 3.95E-04(+) | 1.90E-04(-) | 5.01E-04(+) | 4.16E-04(-) | 2.02E-04(-) |
| PIHS | T1 | 4.38E-04 | 9.13E-04(+) | 4.55E-04(+) | 9.94E-04(+) | 4.72E-04(+) | 2.17E-03(+) | 1.36E-03(+) |
| | T2 | 6.83E-03 | 2.51E-02(+) | 1.48E-02(+) | 3.95E-02(+) | 2.22E-01(+) | 5.89E-01(+) | 5.91E-02(+) |
| PIMS | T1 | 1.86E-03 | 4.11E-03(+) | 2.26E-03(+) | 1.88E-03(+) | 2.39E-02(+) | 5.42E-03(+) | 4.11E-03(+) |
| | T2 | 1.33E+ 01 | 1.53E+ 01(+) | 1.36E+ 01(+) | 2.52E+ 00(-) | 1.52E+ 01(+) | 1.55E+ 01(+) | 1.60E+ 01(+) |
| PILS | T1 | 2.66E-04 | 3.18E-04(+) | 4.57E-04(+) | 3.07E-04(+) | 1.10E-02(+) | 5.25E-04(+) | 2.75E-04(+) |
| | T2 | 5.56E-03 | 1.23E-02(+) | 5.68E-03(+) | 6.39E-02(+) | 1.72E+ 00(+) | 2.96E-02(+) | 6.35E-01(+) |
| NIHS | T1 | 1.33E+ 00 | 1.56E+ 00(+) | 1.52E+ 00(+) | 1.57E+ 00(+) | 4.46E+ 01(+) | 5.38E+ 00(+) | 5.21E+ 01(+) |
| | T2 | 2.42E-04 | 4.71E-04(+) | 4.93E-04(+) | 3.26E-04(+) | 4.68E-04(+) | 4.61E-03(+) | 8.09E-04(+) |
| NIMS | T1 | 1.15E-01 | 4.18E-01(+) | 3.76E-01(+) | 1.88E-01(+) | 1.41E+ 01(+) | 2.81E-01(+) | 4.40E-01(+) |
| | T2 | 6.31E-04 | 2.70E-02(+) | 1.38E-02(+) | 7.27E-04(+) | 1.53E-03(+) | 4.77E-02(+) | 6.44E-02(+) |
| NILS | T1 | 1.07E-02 | 8.33E-04(-) | 1.11E-03(-) | 6.52E-04(-) | 1.67E-02(+) | 9.70E-04(-) | 7.85E-04(-) |
| | T2 | 2.38E-04 | 6.43E-01(+) | 6.42E-01(+) | 4.88E-01(+) | 1.92E+ 01(+) | 6.43E-01(+) | 6.42E-01(+) |
| | | | + 15/− 3/= 0 | + 15/− 3/= 0 | + 13/− 5/= 0 | + 15/− 3/= 0 | + 16/− 2/= 0 | + 16/− 2/= 0 |

**Table 3**
Averaged value and standard deviation of the IGD on the complex MOMTO test suite.

| Problem | Task | HMOMFMA | MOMFEA | MOMFEA-II | EMT-A | MFEA-SADE | MFEA-GHS | NSGA-II |
|---|---|---|---|---|---|---|---|---|
| CPLX1 | T1 | 5.03E-04 | 4.31E-04(-) | 1.87E-02(+) | 2.54E-04(-) | 2.86E-04(-) | 5.37E-04(+) | 5.26E-04(+) |
| | T2 | 8.82E-04 | 5.72E-03(+) | 1.64E-01(+) | 4.62E-03(+) | 1.25E-03(+) | 5.51E-03(+) | 7.07E-03(+) |
| CPLX2 | T1 | 3.54E-04 | 3.90E-04(+) | 1.71E-02(+) | 2.47E-04(-) | 2.78E-04(-) | 4.84E-04(+) | 4.22E-04(+) |
| | T2 | 6.23E-03 | 4.15E-03(-) | 1.70E-01(+) | 1.27E-02(+) | 6.81E-04(-) | 1.22E-02(+) | 1.04E-02(+) |
| CPLX3 | T1 | 1.71E-03 | 5.41E-03(+) | 1.57E-01(+) | 3.68E-03(+) | 1.88E-03(+) | 4.54E-03(+) | 4.94E-03(+) |
| | T2 | 1.81E-03 | 2.97E-03(+) | 1.38E-01(+) | 1.79E-03(-) | 1.41E-03(-) | 2.79E-03(+) | 2.56E-03(+) |
| CPLX4 | T1 | 1.26E-03 | 4.27E-03(+) | 1.64E-01(+) | 6.62E-03(+) | 1.76E-03(+) | 5.27E-03(+) | 6.18E-03(+) |
| | T2 | 1.83E-03 | 4.73E-03(+) | 1.95E-01(+) | 8.03E-03(+) | 2.17E-03(+) | 4.50E-03(+) | 6.73E-03(+) |
| CPLX5 | T1 | 1.93E-03 | 2.32E-03(+) | 1.07E-01(+) | 5.19E-03(+) | 2.19E-03(+) | 2.68E-03(+) | 2.54E-03(+) |
| | T2 | 3.20E-03 | 6.14E-03(+) | 2.84E-01(+) | 1.76E-02(+) | 3.56E-03(+) | 5.93E-03(+) | 6.60E-03(+) |
| CPLX6 | T1 | 1.15E-03 | 2.79E-03(+) | 1.11E-01(+) | 3.92E-03(+) | 2.06E-03(+) | 2.56E-03(+) | 3.90E-03(+) |
| | T2 | 1.83E-03 | 5.18E-03(+) | 2.01E-01(+) | 5.46E-03(+) | 3.31E-03(+) | 4.94E-03(+) | 7.11E-03(+) |
| CPLX7 | T1 | 2.09E-03 | 2.54E-03(+) | 1.47E-01(+) | 2.21E-03(+) | 1.37E-03(-) | 2.57E-03(+) | 2.58E-03(+) |
| | T2 | 9.02E-04 | 1.38E-03(+) | 1.13E-01(+) | 2.26E-03(+) | 1.41E-03(+) | 2.24E-03(+) | 2.16E-03(+) |
| CPLX8 | T1 | 8.75E-04 | 2.74E-03(+) | 1.13E-01(+) | 4.87E-03(+) | 9.22E-04(+) | 2.77E-03(+) | 1.89E-03(+) |
| | T2 | 8.51E-03 | 1.12E-02(+) | 3.55E-01(+) | 7.62E-03(-) | 2.24E-03(-) | 8.58E-03(+) | 9.97E-03(+) |
| CPLX9 | T1 | 3.63E-03 | 6.17E-03(+) | 2.94E-01(+) | 3.21E-03(-) | 4.25E-03(+) | 7.35E-03(+) | 6.26E-03(+) |
| | T2 | 1.21E-03 | 6.04E-03(+) | 1.99E-01(+) | 4.56E-03(+) | 4.09E-03(+) | 5.27E-03(+) | 6.24E-03(+) |
| CPLX10 | T1 | 9.56E-03 | 1.21E-02(+) | 3.18E-01(+) | 8.21E-03(-) | 7.19E-03(-) | 1.15E-02(+) | 1.07E-02(+) |
| | T2 | 8.88E-03 | 9.70E-03(+) | 2.31E-01(+) | 9.67E-03(+) | 9.69E-03(+) | 9.25E-03(+) | 1.08E-02(+) |
| | | | + 18/− 2/= 0 | + 20/− 0/= 0 | + 14/− 6/= 0 | + 13/− 7/= 0 | + 20/− 0/= 0 | + 20/− 1/= 0 |

values over 20 independent runs of each algorithm are demonstrated, and the best result on each sub-problem is marked in gray. The Wilcoxon rank-sum test at the 95% confidence level was applied for the experimental results to compare the proposed HMOMFMA with other comparison algorithms. From Table 3, compared to the state-of-the-art EMT algorithms MOMFEA, MOMFEA-II, EMT-A, MFEA-SADE, and MFEA-GHS, in terms of the IGD metric, the proposed HMOMFMA obtains superior results on 18, 20, 14, 13, and 20 out of 20 sub-problems respectively in the complex MOMTO test suite. Compared to NSGA-II, the proposed HMOMFMA can get better results on all sub-problems.

The proposed HMOMFMA can achieve such good results on the complex MOMTO benchmark test suite because the knowledge transfer method based on decision variable classification can reduce the interference caused by the different types of decision variables transferring knowledge between tasks. The proposed HMOMFMA combines the immune algorithm and EGS as global and local search operators, respectively. The immune algorithm has strong convergence performance and maintains the diversity of the population through clone resource allocation. In contrast, EGS has good local search performance and can accelerate convergence by adapting the mutation length and guiding the evolution according to the gradient information. The collaboration of global search and local search can significantly improve the convergence ability of the algorithm, making HMOMFMA perform best on the complex MOMTO benchmark test suite.

### *4.7. Discussion of proposed strategies*

This section discusses the contribution of each proposed strategy to the performance of the proposed algorithm. Table 4 shows the results of decision variables classification on the classical and complex MOMTO benchmark test suites. Table 5 shows the IGD values of the multi-objective EMT evolution strategy algorithms with different operators on the classical MOMTO benchmark test suite, respectively. The average metric values over 20 independent runs of each algorithm are demonstrated, and the best result for each sub-problem is marked in gray. In addition, the Wilcoxon rank-sum test at the 95% confidence level was applied for the experimental results to compare the proposed HMOMFMA with other comparison algorithms. The significantly better, significantly worse, and not comparable results are represented using "+ ," "-," and "= ," respectively. The MOMFMA only uses the multi-

**Table 4**
The results of decision variables classification on the mainstream benchmarks.

| MTOP | Number of objectives | Number of variables | Diversity-related variables | Convergence-related variables |
|---|---|---|---|---|
| CIHS-T1, CIHS-T2, CILS-T1, CILS-T2, PIHS-T1, PIHS-T2, PIMS-T1, PIMS-T2, PILS-T1, PILS-T2, NIHS-T1, NIHS-T2 | 2 | 50 | $x_1$ | $x_2, \ldots, x_{50}$ |
| CIMS-T1 | 2 | 10 | $x_1$ | $x_2, \ldots, x_{10}$ |
| CIMS-T2 | 2 | 10 | $x_1, x_9$ | $x_2, \ldots, x_8, x_{10}$ |
| NIMS-T1 | 3 | 20 | $x_1, x_2$ | $x_3, \ldots, x_{20}$ |
| NIMS-T2 | 2 | 20 | $x_1, x_2$ | $x_3, \ldots, x_{20}$ |
| NILS-T1 | 3 | 25 | $x_1, x_2$ | $x_3, \ldots, x_{24}$ |
| NILS-T2 | 2 | 50 | $x_1, x_2$ | $x_3, \ldots, x_{50}$ |
| CPLX1-T1, CPLX2-T1 | 2 | 10 | $x_1, x_{10}$ | $x_2, \ldots, x_9$ |
| CPLX1-T2, CPLX3-T1, CPLX3-T2, CPLX4-T1, CPLX4-T2, CPLX5-T1, CPLX6-T1, CPLX6-T2, CPLX7-T1, CPLX7-T2, CPLX8-T1, CPLX9-T2 | 2 | 30 | $x_1$ | $x_2, \ldots, x_{30}$ |
| CPLX2-T2, CPLX8-T2, CPLX10-T1, CPLX10-T2 | 2 | 10 | $x_{10}$ | $x_1, \ldots, x_9$ |
| CPLX5-T2, CPLX9-T1 | 3 | 10 | $x_1$ | $x_2, \ldots, x_{10}$ |

**Table 5**
Averaged value of IGD obtained by MOMFMA, MOMFMA-DVC, MOMFMA-LS, HMOMFMA on the classical MOMTO benchmark test suite.

| Problem | Task | MOMFMA | MOMFMA-DVC | MOMFMA-LS | HMOMFMA |
|---|---|---|---|---|---|
| CIHS | T1 | 1.11E-03(+) | 9.89E-04(+) | 4.20E-04(+) | 3.35E-04 |
| | T2 | 1.39E-03(+) | 9.52E-04(+) | 9.97E-04(+) | 8.64E-04 |
| CIMS | T1 | 1.05E-01(+) | 1.00E-01(+) | 8.94E-02(+) | 4.19E-02 |
| | T2 | 1.07E-02(+) | 7.10E-03(+) | 9.77E-03(+) | 5.72E-03 |
| CILS | T1 | 2.69E-01(+) | 2.37E-01(+) | 2.43E-01(+) | 8.30E-02 |
| | T2 | 3.93E-04(+) | 3.66E-04(+) | 3.56E-04(+) | 3.28E-04 |
| PIHS | T1 | 8.98E-04(+) | 8.72E-04(+) | 7.41E-04(+) | 4.38E-04 |
| | T2 | 2.95E-02(+) | 2.67E-02(+) | 1.08E-02(+) | 6.83E-03 |
| PIMS | T1 | 1.06E-02(+) | 5.39E-03(+) | 4.65E-03(+) | 1.86E-03 |
| | T2 | 4.25E+ 01(+) | 4.14E+ 01(+) | 3.78E+ 01(+) | 1.33E+ 01 |
| PILS | T1 | 2.87E-04(+) | 2.75E-04(+) | 2.69E-04(+) | 2.66E-04 |
| | T2 | 1.00E-02(+) | 6.12E-03(+) | 7.47E-03(+) | 5.56E-03 |
| NIHS | T1 | 5.54E+ 00(+) | 4.78E+ 00(+) | 2.26E+ 00(+) | 1.33E+ 00 |
| | T2 | 9.77E-04(+) | 7.20E-04(+) | 8.49E-04(+) | 2.42E-04 |
| NIMS | T1 | 2.74E-01(+) | 1.31E-01(+) | 1.25E-01(+) | 1.15E-01 |
| | T2 | 3.56E-03(+) | 1.55E-03(+) | 1.07E-03(+) | 6.31E-04 |
| NILS | T1 | 4.02E-02(+) | 2.21E-02(+) | 2.54E-02(+) | 1.07E-02 |
| | T2 | 8.84E-03(+) | 4.73E-03(+) | 4.60E-03(+) | 2.38E-04 |
| | | + 18/− 0/= 0 | + 18/− 0/= 0 | + 18/− 0/= 0 | |

**Table 6**
Averaged value of IGD obtained by HMOMFMA with different *LS* on the classical MOMTO benchmark test suite.

| Problem | Task | HMOMFMA $LS$= 0.01 | HMOMFMA $LS$= 0.02 | HMOMFMA $LS$= 0.05 | HMOMFMA $LS$= 0.1 | HMOMFMA $LS$= 0.2 | HMOMFMA $LS$=0.4 |
|---|---|---|---|---|---|---|---|
| CIHS | T1 | 4.28E-04(+) | 3.35E-04 | 5.18E-04(+) | 5.97E-04(+) | 6.75E-04(+) | 9.18E-04(+) |
| | T2 | 9.59E-04(+) | 8.64E-04 | 1.02E-03(+) | 1.06E-03(+) | 1.40E-03(+) | 5.13E-03(+) |
| CIMS | T1 | 1.53E-01(+) | 4.19E-02 | 1.52E-01(+) | 1.28E-01(+) | 1.14E-01(+) | 1.37E-01(+) |
| | T2 | 1.03E-02(+) | 5.72E-03 | 6.49E-03(+) | 1.29E-02(+) | 1.39E-02(+) | 8.48E-03(+) |
| CILS | T1 | 2.61E-01(+) | 8.30E-02 | 3.25E-01(+) | 2.39E-01(+) | 2.55E-01(+) | 4.45E-01(+) |
| | T2 | 3.75E-04(+) | 3.28E-04 | 4.15E-04(+) | 4.98E-04(+) | 5.83E-04(+) | 6.75E-04(+) |
| PIHS | T1 | 5.94E-04(+) | 4.38E-04 | 8.07E-04(+) | 9.78E-04(+) | 3.83E-03(+) | 2.95E-03(+) |
| | T2 | 1.18E-02(+) | 6.83E-03 | 2.11E-02(+) | 1.70E-02(+) | 4.29E-02(+) | 1.10E-01(+) |
| PIMS | T1 | 8.02E-03(+) | 1.86E-03 | 8.74E-03(+) | 7.85E-03(+) | 1.47E-02(+) | 1.44E-02(+) |
| | T2 | 3.55E+ 01(+) | 1.33E+ 01 | 3.75E+ 01(+) | 3.32E+ 01(+) | 3.82E+ 01(+) | 3.67E+ 01(+) |
| PILS | T1 | 2.95E-04(+) | 2.66E-04 | 2.96E-04(+) | 3.38E-04(+) | 3.40E-04(+) | 9.48E-04(+) |
| | T2 | 5.95E-03(+) | 5.56E-03 | 6.40E-03(+) | 7.79E-03(+) | 1.05E-02(+) | 9.59E-03(+) |
| NIHS | T1 | 3.43E+ 00(+) | 1.33E+ 00 | 3.75E+ 00(+) | 4.32E+ 00(+) | 4.85E+ 00(+) | 4.58E+ 00(+) |
| | T2 | 6.05E-04(+) | 2.42E-04 | 5.86E-04(+) | 9.02E-04(+) | 1.84E-03(+) | 2.70E-03(+) |
| NIMS | T1 | 1.29E-01(+) | 1.15E-01 | 1.53E-01(+) | 2.29E-01(+) | 2.56E-01(+) | 2.46E-01(+) |
| | T2 | 1.67E-03(+) | 6.31E-04 | 3.07E-03(+) | 2.04E-03(+) | 1.79E-03(+) | 6.77E-03(+) |
| NILS | T1 | 1.07E-02(=) | 1.07E-02 | 1.07E-02(=) | 1.06E-02(-) | 1.07E-02(=) | 1.08E-02(+) |
| | T2 | 2.54E-04(+) | 2.38E-04 | 2.56E-04(+) | 2.52E-04(+) | 2.55E-04(+) | 2.58E-04(+) |
| | | + 17/− 0/= 1 | | + 17/− 0/= 1 | + 17/− 1/= 0 | + 17/− 0/= 1 | + 18/− 0/= 0 |

**Table 7**
Averaged value of IGD obtained by HMOMFMA with different *r* on the classical MOMTO benchmark test suite.

| Problem | Task | HMOMFMA $r = 0.05$ | HMOMFMA $r = 0.1$ | HMOMFMA $r = 0.2$ | HMOMFMA $r = 0.5$ | HMOMFMA $r = 0.8$ |
|---|---|---|---|---|---|---|
| CIHS | T1 | 3.35E-04 | 5.59E-04(+) | 5.11E-04(+) | 6.33E-04(+) | 7.25E-04(+) |
| | T2 | 8.64E-04 | 9.98E-04(+) | 9.52E-04(+) | 9.53E-04(+) | 1.39E-03(+) |
| CIMS | T1 | 4.19E-02 | 1.46E-01(+) | 1.22E-01(+) | 1.08E-01(+) | 1.36E-01(+) |
| | T2 | 5.72E-03 | 1.03E-02(+) | 8.61E-03(+) | 7.75E-03(+) | 7.54E-03(+) |
| CILS | T1 | 8.30E-02 | 2.69E-01(+) | 3.02E-01(+) | 2.98E-01(+) | 4.28E-01(+) |
| | T2 | 3.28E-04 | 3.93E-04(+) | 3.88E-04(+) | 4.09E-04(+) | 4.36E-04(+) |
| PIHS | T1 | 4.38E-04 | 5.98E-04(+) | 7.91E-04(+) | 7.08E-04(+) | 8.63E-04(+) |
| | T2 | 6.83E-03 | 1.02E-02(+) | 8.38E-03(+) | 1.84E-02(+) | 3.12E-02(+) |
| PIMS | T1 | 1.86E-03 | 7.68E-03(+) | 7.72E-03(+) | 9.04E-03(+) | 1.01E-02(+) |
| | T2 | 1.33E+ 01 | 4.26E+ 01(+) | 2.98E+ 01(+) | 3.81E+ 01(+) | 2.91E+ 01(+) |
| PILS | T1 | 2.66E-04 | 2.91E-04(+) | 2.73E-04(+) | 2.88E-04(+) | 2.85E-04(+) |
| | T2 | 5.56E-03 | 6.71E-03(+) | 6.12E-03(+) | 7.47E-03(+) | 1.00E-02(+) |
| NIHS | T1 | 1.33E+ 00 | 3.35E+ 00(+) | 4.10E+ 00(+) | 4.87E+ 00(+) | 4.76E+ 00(+) |
| | T2 | 2.42E-04 | 6.62E-04(+) | 7.14E-04(+) | 9.10E-04(+) | 1.06E-03(+) |
| NIMS | T1 | 1.15E-01 | 2.63E-01(+) | 1.88E-01(+) | 1.80E-01(+) | 1.33E-01(+) |
| | T2 | 6.31E-04 | 1.31E-03(+) | 1.18E-03(+) | 1.22E-03(+) | 1.55E-03(+) |
| NILS | T1 | 1.07E-02 | 1.07E-02(=) | 1.07E-02(=) | 1.07E-02(=) | 1.05E-02(-) |
| | T2 | 2.38E-04 | 2.65E-04(+) | 2.47E-04(+) | 2.72E-04(+) | 2.47E-04(+) |
| | | | + 17/− 0/= 1 | + 17/− 0/= 1 | + 17/− 0/= 1 | + 17/− 1/= 0 |

objective multifactorial immune algorithm. MOMFMA-DVC applies the evolutionary recombination strategy based on decision variables' characteristics to MOMFMA. MOMFMA-LS hybrids the EGS as the local search operator to MOMFMA. The parameters of the comparison algorithm are consistent. It can be seen from Table 5 that the algorithm results of MOMFMA-DVC and MOMFMA-LS are better than MOMFMA in 18 sub-problems. This proves that the evolutionary recombination strategy based on decision variables' characteristics and EGS as a local search operator are valid. And the combination of the two strategies can greatly improve the performance of the algorithm.

### 4.8. Parameter sensitivity analysis

In order to ensure that the parameter settings used by the proposed algorithm are reasonable, in this section, the parameters *LS* and *r* involved in the local search operator of the algorithm are analyzed. A comparative experiment on the classical multi-objective multi-task optimization benchmark test suite is conducted. The average values over 20 independent runs of each algorithm are demonstrated, and the best result for each sub-problem is marked in gray. In addition, the Wilcoxon rank-sum test at the 95% confidence level was applied for the experimental results, where significantly better, significantly worse, and not comparable are indicated using "+ ", "-", and "= ", respectively. The IGD values of the classical MOMTO benchmark test suites are listed in Table 6 and Table 7, respectively.

## 5. Conclusion

This paper proposes a novel evolutionary multitasking algorithm for multi-objective optimization, namely HMOMFMA, by hybridizing the decision variable classification method, multi-objective immune algorithm, and evolutionary gradient search. The decision variable classification method is applied to classify the decision variables into convergence and diversity-related decision variables. Unique strategies will be used for different decision variables during recombination and information transfer. This can bring benefits for improving information transfer efficiency in multitasking optimization and accelerating convergence. The immune algorithm focuses on the most sparsely distributed nondominated solutions in the population, has strong convergence ability, and can guarantee the diversity of the population. The evolutionary gradient search method is introduced as the local search operator, which can accelerate convergence by adaptively adjusting the mutation length and guiding the evolution according to the gradient information. Comprehensive experiments are conducted on both the classical and complex MOMTO test suites. The proposed HMOMFMA is compared with five state-of-the-art multi-objective EMT algorithms, MOMFEA [35], MOMFEA-II [26], EMT-A [32], MFEA-SADE [36], and MFEA-GHS [37] and a classic MOEA, namely NSGA-II [6]. The experimental results demonstrate that the proposed HMOMFMA is superior to other advanced EMT algorithms.

However, some issues can still be considered in future work. First, the proposed HMOMFMA can extend to optimize three or more tasks simultaneously. The modules that can select the most appropriate task for transferring from multiple tasks should be designed. Second, the method of classifying decision variables can be improved, and dynamic classification can also be considered during the process of evolutionary iteration. Next, the proposed HMOMFMA mainly solves general MOPs. The improved HMOMFMA can also solve expensive MOPs, dynamic MOPs, large-scale MOPs, multimodal MOPs, and other more complex problems. Finally, the information transfer method in the proposed HMOMFMA can be further improved to reduce the negative transfer.

## CRediT authorship contribution statement

**xu zhiwei:** Methodology, Project administration, Writing – original draft, Writing – review & editing. **Zhang Kai:** Methodology, Validation. **Xu Jiafeng:** Data curation. **Xu Xin:** Methodology, Validation. **Wu Ni:** Software, Validation, Visualization. **He Juanjuan:** Methodology, Software.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

Data will be made available on request.

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