Solving Multimodal Multi-Objective Problems with Local Pareto Front using a Population Clustering Mechanism

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ABSTRACT

Most existing multimodal multi-objective evolutionary algorithms only search the global Pareto front of the problem while ignoring the excellent local Pareto front of the problem. To address this issue, an optimization algorithm with population clustering mechanism is proposed to settle multimodal multi-objective problems with local Pareto front. At the first step, a partitioning method is used to divide the total population into main rank and other ranks and a population clustering method is proposed to repartition the entire population into global Pareto front subpopulations and local Pareto front subpopulations. In the second step, each subpopulation evolves independently and the diversity in the objective space and decision space are considered simultaneously. An improved density adaptive adjustment strategy is put forward to enhance the diversity of the population in the decision space. In the experimental part, the algorithm is compared with several other state-of-the-art algorithms using the CEC 2019 MMOPs test case, and the result of the experiment confirm that the algorithm proposed shows excellent performance.

CCS CONCEPTS

• **Theory of computation** → Design and analysis of algorithms; Mathematical optimization; Continuous optimization; Bio-inspired optimization.

KEYWORDS

multimodal multi-objective optimization, population clustering, local Pareto front, multimodal multi-objective evolutionary algorithm

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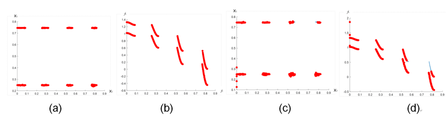

Figure 1: Individuals are more evenly distributed after adding main rank decision space density adjustment and diversity adjustment. (MMF12) (a) is the decision space (b) is the objective space. Results density adjustment strategy of [9] run on MMF12 (c) is the decision space (d) is the objective space, the global PF distribution and diversity is excellent, but the local PF is weak with respect to the distribution and convergence

1 INTRODUCTION

Multimodal Multi-objective problems (MMOPs) mean that there are two or more Pareto sets (PS) corresponding to the same Pareto front (PF) in a problem. The multimodal multi-objective evolutionary algorithms (MMOEAs) are raised to resolve the MMOPs. Many MMOEAs have been put forward to handle MMOPs with global PF. But they cannot give satisfactory results for MMOPs with local PF.

In real world, for problems with global PF and local PF, the local PF can be the choice of the decision maker when the global PF is not available as an option for reasons such as accidental or realistic unattainability. Therefore, for practical problems, it is important and necessary to get both the global PF and the local PF.

In order to handle the problem mentioned above, a population clustering multimodal multi-objective evolutionary algorithm (PC-MMOEA) is presented. First, the global PF and the local PF of the problem are found by local exploration. Second, the population is divided and the global PF and the local PF are separated. Then, the subpopulations evolve independently without being influenced by other subpopulations to ensure that the global PF will not eliminate the local PF.

The major contributions of the paper are listed below. First, a population clustering mechanism is proposed. The selection process is restricted in the respective rank, which can avoid the local PF from being dominated by global PF. The second one is an improved

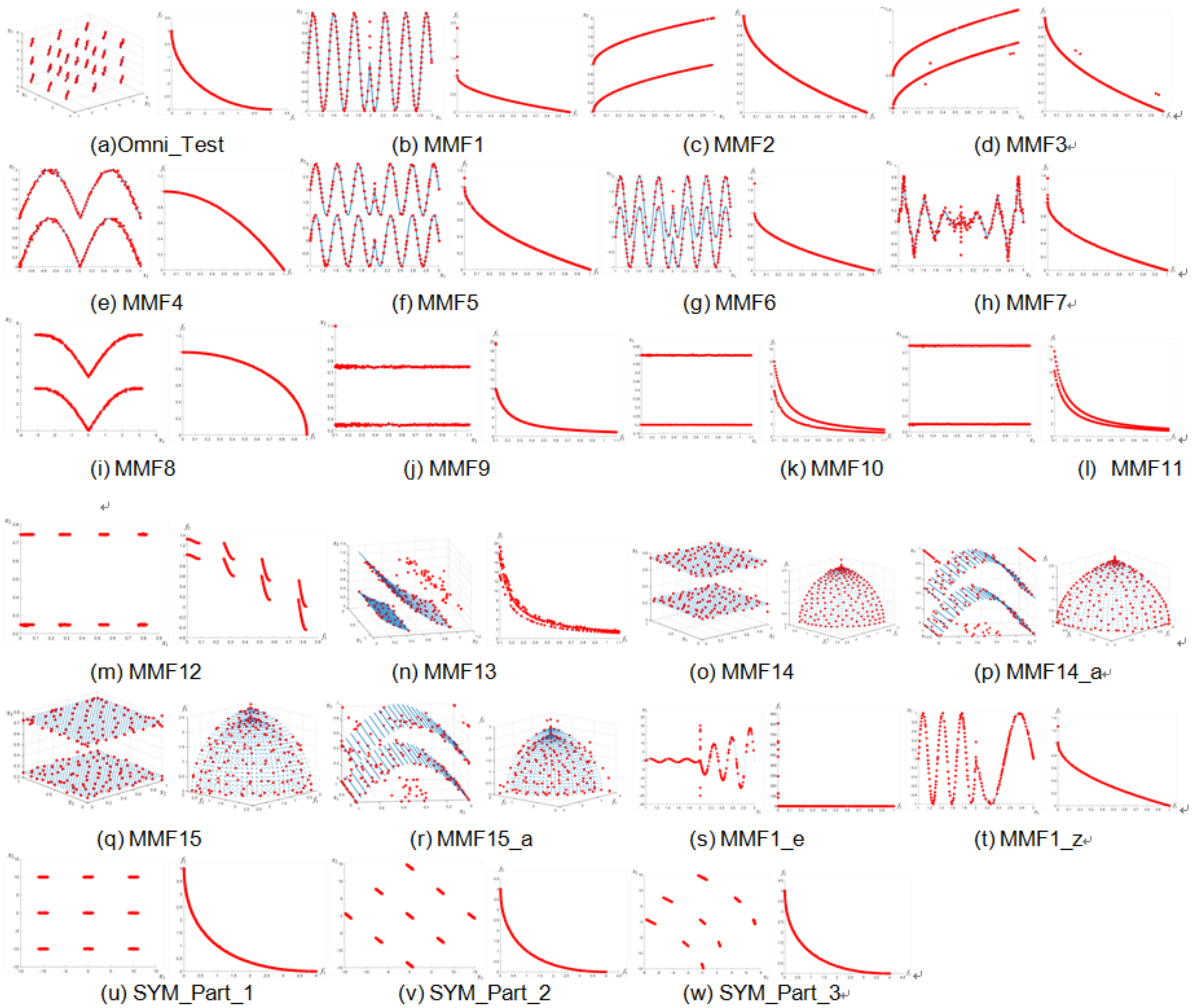

Figure 2: The result of PC-MMOEAs runs on the 23 MMOPs

density adaptive adjustment strategy, which improved population diversity of decision space

The rest of this paper mainly is made up by the sections below, the second section introduces the related works of MMOPs and MMOEA. Section three elaborates the algorithm proposed with detailed descriptions. The forth section present the experiments and the data, and the comparison between the proposed algorithm and other state-of-the-art algorithms is shown. The fifth section gives a summary of the whole paper and gives the future work.

2 RELATED WORKS AND MOTIVATION

The MMOPs can be briefly summed up in the following formula:

$$\min_{x \in R} F(x) = (f_1(x), \ldots, f_n(x))$$

Where $\mathbf{x}$=$(x_1, \ldots, x_m)$represents the value of the decision space, m stands for the number of decision variables, R is the decision space. $f_1(x), \ldots, f_n(x)$ represents the values of the objective functions, n is the number of objective functions. The PFs of different MMOPs are diverse. Some problems only have the global PF, while others carry both the global PF and the local PF.

However, there are several difficulties to solve MMOPs with both local PF and global PF. (1) good convergence of the solution set in the objective space; (2) excellent diversity of the solution set in the objective space; (3) excellent diversity of the solution set in the decision space.

2.1 Multimodal multi-objective optimization approaches

2.1.1 Non-dominated sorting-based Approaches. These algorithms are based on non-dominated sorting and have good convergence. Preferentially selecting individuals with lower non-dominated ranking and adding other criteria in environment selection to improve

Table 1: Average IGD values over 30 runs on CEC 2019 MMOPs, where the best mean is shown in a GRAY background

Problem	MMOCLRPSO	CPDEA	MOEADAD	MO_Ring_PSO_SCD	NIMMO	TriMOEATAR	PC-MMOEA
MMF1	3.2043e-3-	2.4131e-3+	6.5547e-3-	4.0051e-3-	5.1367e-3-	3.5960e-3-	2.7065e-3
MMF2	5.1888e-3-	3.4217e-3-	1.5792e-2-	1.0015e-2-	1.1040e-2-	4.9598e-3-	3.2281e-3
MMF3	5.6470e-3-	4.4043e-3-	1.7710e-2-	8.7345e-3-	1.0635e-2-	9.7236e-3-	4.2808e-3
MMF4	3.0935e-3-	2.4703e-3-	3.1974e-3-	3.2386e-3-	5.2859e-3-	9.6222e-3-	2.4294e-3
MMF5	3.0584e-3-	2.3757e-3+	5.9265e-3-	3.6278e-3-	3.7961e-3-	4.6401e-3-	2.7097e-3
MMF6	3.0544e-3-	2.3432e-3+	5.6887e-3-	3.6755e-3-	3.5817e-3-	3.7254e-3-	2.6394e-3
MMF7	3.0462e-3=	2.5489e-3+	3.5685e-3-	3.6054e-3-	2.8192e-3+	4.0787e-3-	3.0635e-3
MMF8	4.6183e-3-	2.5795e-3-	1.1529e-2-	3.6255e-3-	1.5188e-2-	9.5289e-3-	2.5246e-3
MMF9	1.5678e-2-	9.9119e-3=	1.7142e-2-	1.4716e-2-	1.1171e-2-	7.3682e-2-	1.0254e-2
MMF10	1.6498e-1-	1.9425e-1-	1.8574e-1-	1.4825e-1-	4.2077e-2-	2.3221e-1-	1.9937e-2
MMF11	8.6626e-2-	9.3135e-2-	9.1792e-2-	8.6466e-2-	1.8396e-2+	1.6853e-1-	2.0789e-2
MMF12	5.6760e-2-	8.2745e-2-	8.4736e-2 -	6.1752e-2-	7.8690e-3-	8.5102e-2-	3.8050e-3
MMF13	1.2529e-1-	1.4251e-1-	1.4139e-1-	1.1087e-1-	4.3228e-2-	2.4991e-1-	3.5981e-2
MMF14	9.7823e-2-	6.9648e-2+	8.0390e-2-	1.0625e-1-	1.0754e-1-	8.9598e-2-	7.3773e-2
MMF15	9.6821e-2-	7.2449e-2+	8.7394e-2-	1.0096e-1-	1.2646e-1-	1.0083e-1-	7.9567e-2
MMF14_a	1.8897e-1-	1.8032e-1-	1.9443e-1-	1.9485e-1-	1.9526e-1-	2.0879e-1-	1.1905e-1
MMF15_a	1.9109e-1-	1.7689e-1-	1.9139e-1-	1.9272e-1-	2.0116e-1-	1.9989e-1-	1.4683e-1
MMF1_e	3.8652e-3-	2.7235e-3+	1.1335e-1-	3.5982e-3-	4.5691e-3-	3.7295e-3-	2.8691e-3
MMF1_z	3.0877e-3-	2.3667e-3=	6.3742e-3-	3.7955e-3-	4.1645e-3-	3.8215e-3-	2.3938e-3
Omni_test	2.2388e-2-	7.6483e-3+	1.2503e-2-	1.6396e-2-	4.3128e-2-	1.5085e-2-	9.4235e-3
SYM_PART_1	1.5727e-2-	9.3507e-3+	2.6702e-2-	1.3311e-2-	2.6815e-2-	4.8660e-2-	1.0370e-2
SYM_PART_2	1.7551e-2-	1.0775e-2-	1.9383e-2-	1.2998e-2-	3.3515e-2-	4.0289e-2-	9.9339e-3
SYM_PART_3	2.4128e-2-	1.1076e-2=	2.0962e-2-	1.4677e-2-	3.2889e-2-	8.4595e-2-	1.0821e-2
+/-/=	0/22/1	9/11/3	0/23/0	0/23/0	2/21/0	0/23/0	

the diversity of decision space and objective space. For example, DN-NSGA-II [1], Omni-Optimizer [2], MMOCLRPSO [3] and MO-Ring-PSO-SCD [4]. However, the local PF is dominated by the global PF in environment selection, resulting in the local PF being in a relatively inferior rank after the ranking, and eventually eliminated.

2.1.2 Decomposition-based Approaches. These algorithms divide the multi-objective multimodal problem into multiple sub-problems and collaboratively optimize each sub-problem, which has a great advantage in the diversity of the objective space. Such as MOEA/D-AD [5], Tri-MOEA-TA&R [6].

Nevertheless, such algorithms take the non-dominated solutions as the optimization goal in the optimization process, and the local PF individuals is dominated by the non-dominated individuals, which is often discarded during the iteration process.

2.1.3 Indicator-based approaches. These algorithms filter and evolve by computing different metrics with good convergence and diversity. For example, CPDEA [7] uses local convergence quality rather than global convergence quality, and uses a convergence density penalty strategy to make individuals uniformly distributed in the decision space. Each individual of NIMMO [8] only compares with its T nearby individuals in terms of fitness to remove the worst individual, so that the individual found is the optimal individual in the current local space. And the niching mechanism is added to the decision space to improve the diversity of the decision space.

3 PROPOSED ALGORITHM

This section describes in detail PC-MMOEA, an algorithm that preserves the local PF and global PF of MMOPs and is able to obtain population with good diversity and convergence.

3.1 Population clustering

To solve the problems mentioned above, the concept of main rank is proposed. After the non-dominated sorting, if the individuals of a certain rank exceed 20% of the individuals of the whole population, then the rank is identified as an important rank, which is the main rank. As for the other individuals, firstly, the non-dominated sorting is carried out to divide the main rank, and for the individuals in the non-main rank, they are divided into the nearest main rank based on the distance between this individual and the individuals in the main rank in the decision space, and their rank is replaced with the rank of the main rank. According to this algorithm, all the individuals in the population are classified into in main rank. The specific operation is described in ALGORITHM 1, all distance mentioned in this paper means Euclidean distance.

3.2 Improved density adjustment strategy

The problem of density unevenness is mentioned in the [9] and an adaptive density adjustment strategy is presented to improve the density inequality problem in the population. However, the strategy

Table 2: Average IGDx values over 30 runs on CEC 2019 MMOPs, where the best mean is shown in a GRAY background

Problem	MMOCLRPSO	CPDEA	MOEADAD	MO_Ring_PSO_SCD	NIMMO	TriMOEATAR	PC-MMOEA
MMF1	4.7288e-2-	3.1882e-2-	6.7951e-2-	8.2035e-2-	5.7642e-2-	3.7416e-2-	3.0593e-2
MMF2	9.3179e-3-	4.8577e-3-	4.0192e-2-	3.5023e-2-	6.6257e-2-	4.2562e-2-	4.4720e-3
MMF3	1.0450e-2-	4.7804e-3-	4.0394e-2-	2.1384e-2-	6.2702e-2-	2.1731e-2-	4.6365e-3
MMF4	2.7319e-2-	1.8850e-2=	1.7698e-2+	1.1044e-1-	4.5975e-2-	4.1148e-2-	1.8905e-2
MMF5	7.9662e-2-	5.9637e-2-	8.2187e-2-	1.6063e-1-	7.6405e-2-	8.9127e-2-	5.6945e-2
MMF6	7.1337e-2-	5.4627e-2 -	8.5396e-2-	1.3779e-1-	8.9233e-2-	7.5762e-2-	5.3543e-2
MMF7	2.2919e-2 -	1.8868e-2+	2.9253e-2-	8.6272e-2-	1.7857e-2+	2.3658e-2-	2.1198e-2
MMF8	5.9340e-2-	3.7454e-2-	1.6258e-1-	3.3891e-1-	1.8491e-1-	5.8583e-1-	3.6217e-2
MMF9	7.5911e-3-	6.3545e-3-	1.7530e-2-	2.1718e-2-	3.5223e-3+	3.3446e-3+	4.6938e-3
MMF10	1.6505e-1-	2.0092e-1-	1.8865e-1 -	1.7140e-1-	1.9306e-2-	2.0154e-1-	2.8913e-3
MMF11	2.0666e-1-	2.4894e-1-	2.3050e-1-	2.3976e-1-	5.1729e-3-	2.5251e-1-	4.8341e-3
MMF12	1.8992e-1-	2.4507e-1-	2.4545e-1-	2.3323e-1-	3.4752e-3-	2.4819e-1-	2.9921e-3
MMF13	2.4573e-1-	2.5297e-1-	2.7090e-1-	2.5438e-1-	1.4459e-1-	2.6864e-1-	6.2046e-2
MMF14	6.8979e-2-	4.6266e-2-	4.8750e-2-	8.9074e-2-	6.9036e-2-	3.6821e-2+	4.5419e-2
MMF15	8.1617e-2-	6.2590e-2-	7.3557e-2-	1.1129e-1-	1.1915e-1-	7.0352e-2-	6.1743e-2
MMF14_a	1.7993e-1-	2.2467e-1 -	2.5748e-1-	2.2285e-1-	1.6470e-1-	2.7109e-1-	4.8361e-2
MMF15_a	1.9254e-1-	2.0522e-1-	2.1748e-1-	2.1422e-1-	2.1655e-1-	2.2254e-1-	7.5708e-2
MMF1_e	1.9145e-1-	1.3370e-1-	1.7604e+0-	5.7996e-1-	1.1225e+0-	1.9584e+0-	1.2393e-1
MMF1_z	3.4109e-2-	2.4728e-2-	5.3757e-2-	7.9509e-2-	3.7083e-2-	3.3524e-2-	2.1998e-2
Omni_test	1.8079e-1-	7.1986e-2-	1.1338e-1-	7.7717e-1-	4.0269e-1-	8.6840e-1-	5.5681e-2
SYM_PART_1	7.4228e-2-	3.1798e-2+	3.2759e-2+	3.7198e-1 -	4.5976e-2=	2.8526e-2+	4.4781e-2
SYM_PART_2	8.2497e-2-	5.7014e-2-	3.9099e-2+	1.2719e-1-	6.5804e-2-	3.2246e+0-	4.6437e-2
SYM_PART_3	2.7884e-1-	4.7910e-2-	3.0342e-2+	6.3692e-2-	2.1394e-1-	2.5928e+0-	3.7455e-2
+/-/=	0/23/0	2/20/1	4/19/0	0/23/0	2/20/1	3/20/0	

Wilcoxon rank sum test at 0.05 significance level between PC-MMOEA and the compared MMOEA." -" indicates that PC-MMOEA outperforms the compared algorithm, "+" indicates that the compared algorithm outperforms PC-MMOEA, and "=" indicates that there is no comparable.

mentioned above takes no account of local PF and no information about the individuals around the least dense individual is used during the mutation process, moreover, random mutation will lead to poor convergence of the population.

To address the issues proposed above, this paper makes the following improvements: still finding the densest individual from the whole decision space, but finding the least dense individual from the main rank. Since main rank contains both local PF and global PF, it can keep the diversity of the decision space while preserving the local PF. And we use the information of the individuals around the lowest density individual to perform the differential variation, which impacts less on the convergence of the population compared with the original strategy. The relevant operations are shown in ALGORITHM 2, δ is a random value between 1 and 10.
The following figure shows the comparison between density adaptive adjustment in [9] and the main Rank decision space density self-adaptive strategy.

3.3 Overall algorithm

The algorithm proposed in this paper is organized mainly in two steps.

Step I: convergence of populations and adjustment of decision space diversity, the critical operations are as follows: 1: restrict the selection process between itself and its offspring; 2: the calculation of MED is restricted within the same Rank 3: density adjustment is performed in the main rank.

Then, based on the result of population evolution in step I, the population is clustered and the Rank is obtained, after which the population is classified into one or more sub-populations according to that Rank.

Step II: convergence population and adjustment of decision space and objective space diversity; the key operations are as follows: 1: limiting the selection process in its own subpopulation; 2: the MED and MEDx computation is confirmed within the subpopulation. 3: improved adaptive adjustment of density based on main rank.

In the problem with local PF and global PF, local PF is preserved due to the fact that individuals located in local PF are not influenced by the individuals in global PF, not only the population convergence but decision space and objective space diversity are excellent as well. The algorithm introduces the diversity adjustment strategies MED and MEDx proposed in [9],($\boldsymbol{DCount_l}(NewP_t^{(i)})$the number of individuals that dominate $NewP_t^{(i)}$, and the subscript l means that

Algorithm 1 Population clustering

```
Input: Population
Output: New Population with new Rank
set all values in Rank = 0;
set temp = Population;
set i = 0;
set R = the mutation distance of individuals in decision space;
while (i <= Populations' size)and(temp's size > 0)
   Rank rest = NDSort (temp);
   for(individuals in Rank rest = 1 j = 1 : n )
      dist = the distance between individual( j ) and other
individuals in temp in decision space;
      keep all individuals whose dist < R in circle;
      if( size (individuals in circle whose Rank rest =
1))>(size(circle)*20%)
         the Rank of all individuals in circle = i;
      else
         the Rank of all individuals in circle = 0;
      end if
      delete individuals in circle from temp;
      i++;
   end for
end while
for i = 1 : Max(Rank)
   if(size(individuals whose Rank = i )>size(Population)*20%) or (i
= 1)
      add Rank = i to main Rank
   else
      set All individual whose Rank = i to 0;
   end if
end for
Population else = the individuals whose Rank = 0;
for individuals in Population else j = 1 : n
   dist else = the distance between individual( j ) and individuals in
main Rank in decision space;
   minId = get the cloest individual in main Rank;
   Population else( j )= GaussMutation (main Rank (minId));
end for
end
```

all calculations are performed in the main Rank of the individual, the overall framework of the algorithm is shown in ALGORITHM 3.

4 EXPERIMENTAL RESULTS

To verify the effectiveness of the proposed algorithm, PC-MMOES will be in comparison with six State-of-the-art MMOEAs, including MMOCLRPSO [3], CPDEA [7], MOEA/D-AD [5], MO-Ring-PSO-SCD [4], NIMMO [8], and Tri-MOEA-TA&R [6]. The test problems used is CEC 2019 MMOPs [10]. All the test data are carried out on Platemo [11]. The number of population is set to 200 each run has a total of 1000 generations. The DEMutation range used by PC-MMOEA varies according to the size of the decision space, with each dimension varying by 0.2 and -0.2 of the difference between the maximum and minimum values of that dimension. The test indicators, including IGD and IGDx, represent the convergence and diversity of the objective space and the decision space, respectively. And the parameters in the compared state-of-the-art algorithm is set as suggested in original paper. The following tables show PC-MMOEA excellent performance. Out of 23 test problems, PC-MMOEA obtained the optimal values from the IGDx values for 16 problems. Based on the values of IGD, PC-MMOEA obtained the optimal values for 8 problems, and the performance of 2 of them is the same as the optimal values. It can be seen from the pictures below that PC-MMOEA is effective and efficient, not only can well solve MMOPs with local PF and global PF but MMOPs with only global PF.

Algorithm 2 Main Rank Decision Space Density Self-Adaptive Strategy

```
Input: Population
Output: New Population with main rank decision density
self-adaptive
set Radius = The difference between the maximum and minimum
values of the decision space/δ;
for individuals in population i = 1 : n
 dist = the distance between individual ( i ) and other individuals in
Population;
   Count[ i ] = the number of individuals whose dist <Radius;
end for
MaxId = the individuals with the largest value in Count;
non-dominated sorting and calculate main Rank;
MinId = the individuals with the minimum value of Count in main
Rank;
DistMaxId = the minimum distance between MaxId and other
individuals in population in decision space;
DistMinId = the minimum distance between MinId and other
individuals in population in decision space;
if Count[MaxId]/Count[MinId]>1.1 and DistMaxId/DisMinId >1
   if exist the farthest individual within MinOne's mutation range
      MaxId=MinId+(0.5+0.5*rand)(individual-MinId)
   else if exist the two or more individuals in the MinId's rank
      MaxOne= DEMutation (MinOne,individual1,individual2);
   else if there no individuals in MinId's rank
      MaxId=GuassMutation (MinId);
   end if
end if
```

5 CONCLUSION

A population clustering multimodal multi-objective evolutionary algorithm for solving MMOPs is presented in this paper, which can solve MMOPs with local PFs and global PFs well. The superiority of the algorithm in this paper is proved by experiments in test functions. Also developing evolutionary algorithms on large-scale MMOPs will be the direction of our further research.

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Algorithm 3 Proposed PC-MMOEA Algorithm

Initialization P_t , t =0
while(t< 1/2 maximum generation)//step I
$NewP_t = GaussMutation(P_t)$
calculate main Rank
for i =1 : P
if ($NewP_t^{(i)} \prec P_t^{(i)}$)
$P_t^{(i)} = NewP_t^{(i)}$;
else if (t<1/4 maximum generation)
if ($NewP_t^{(i)} \nprec P_t^{(i)}$) and ($P_t^{(i)} \nprec NewP_t^{(i)}$)
if ($\boldsymbol{DCount_l}(NewP_t^{(i)}) < \boldsymbol{DCount_l}(P_t^{(i)})$)
$P_t^{(i)} = NewP_t^{(i)}$;
else if ($\boldsymbol{DCount_l}(NewP_t^{(i)}) = \boldsymbol{DCount_l}(P_t^{(i)})$) and $\frac{\boldsymbol{MED_{xl}}(NewP_t^{(i)})}{\boldsymbol{MED_{xl}}(P_t^{(i)})} > 1$
$P_t^{(i)} = NewP_t^{(i)}$;
end if
end if
end if
end for
if (t<1/4 maximum generation)
main rank decision space density self-adaptive strategy ();
end if
end while
group = population clustering algorithm;
while (t < maximum generation)//step II
for k = 1 : num(group)
$subP_t = P_t(k)$;
for i : $subP^k$
$NewsubP_t^{(i)} = DEMutation(subP_t^{(i)})$;
if ($NewsubP_t^{(i)} \prec subP_t^{(i)}$)
$subP_t^{(i)} = NewsubP_t^{(i)}$
else if ($NewsubP_t^{(i)} \nprec subP_t^{(i)}$) and ($subP_t^{(i)} \nprec NewsubP_t^{(i)}$)
if($\boldsymbol{DCount_l}(NewsubP_t^{(i)}) < \boldsymbol{DCount_l}(subP_t^{(i)})$)
$subP_t^{(i)} = NewsubP_t^{(i)}$;
else if ($\boldsymbol{DCount_l}(NewsubP_t^{(i)}) = \boldsymbol{DCount_l}(subP_t^{(i)})$)
if (t<3/4 maximum generation)
if $\frac{\boldsymbol{MED_{xl}}(NewsubP_t^{(i)})}{\boldsymbol{MED_{xl}}(subP_t^{(i)})} > 1$;
$subP_t^{(i)} = NewsubP_t^{(i)}$;
end if
else
if ($\frac{\boldsymbol{MED_{xl}}(NewsubP_t^{(i)})}{\boldsymbol{MED_{xl}}(subP_t^{(i)})} > 1$ and $\frac{\boldsymbol{MED_l}(NewsubP_t^{(i)})}{\boldsymbol{MED_l}(subP_t^{(i)})} > 1$)
$subP_t^{(i)} = NewsubP_t^{(i)}$;
end if
end if
end if
end if
end for
end for
if (t<3/4 maximum generation)
main rank decision space density self-adaptive strategy ();
end if
end while

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