2021 4th International Conference on Intelligent Autonomous Systems (ICoIAS) | 978-1-6654-4195-7/21/$31.00 ©2021 IEEE | DOI: 10.1109/ICoIAS53694.2021.00043

Multi-population Evolutionary Algorithm for Multimodal Multobjective Optimization

1st Kai Zhang
School of Computer Science and Technology
Wuhan University of Science and Technology
Wuhan, China
zhangkai@wust.edu.cn

2nd Fang Liu
School of Computer Science and Technology
Wuhan University of Science and Technology
Wuhan, China
1213553227@qq.com

3rd Chaonan Shen
School of Computer Science and Technology
Wuhan University of Science and Technology
Wuhan, China
449777215@qq.com

4th Zhiwei Xu
School of Computer Science and Technology
Wuhan University of Science and Technology
Wuhan, China
1786546913@qq.com

***Abstract*—In recent years, numerous efficient and effective multimodal multi-objective evolutionary algorithms (MMOEAs) have been developed to address multimodal multi-objective optimization problems (MMOPs) involving multiple equivalent sets of Pareto optimal solutions to be found simultaneously. However, the Pareto optimal solutions may have various contracting or expending shapes, and have random locations in the decision space. In addition, uniform decision distribution does not imply good objective distribution. Therefore, many existing MMOEAs are very difficult to guide the individuals converged to every Pareto subregion with good distribution in both the decision space and the objective space. In this paper, we present a multi-population evolutionary algorithm to search for the equivalent global Pareto optimal solutions. The original population should be divided into two groups of subpopulations with equal size. The first subpopulation is designed to search for the optimal solutions in objective space. At the same time. the second subpopulation focus to obtain high-quality optimal solutions in the decision space. The multi-population strategy is helpful to improve the decision and objective distributions simultaneously, and address the MMOPs effectively. The proposed algorithm is compared against five state-of-the-art MMOEAs. The experimental results indicate the proposed algorithm provides better performance than competing MMOEAs on IEEE CEC 2019 MMOPs test suite.**

Keywords—multimodal multi-objective optimization problem, multimodal multi-objective evolutionary algorithm, evolution strategy

I. Introduction

For the past few years, many multimodal multi-objective evolutionary algorithms (MMOEAs) have been developed to address multimodal multi-objective optimization problems (MMOPs) involving multiple equivalent sets of Pareto optimal solutions [1]-[3]. The mathematical model of the multi-objective optimization problem (MOP) can be formulated as follows:

$$\min f(x) = \min[f_1(x), f_2(x), \dots f_i(x), \dots, f_M(x)]^T \tag{1}$$

where the decision vector x consists of N decision variables, $x = [x_1, x_2, \dots, x_i, \dots, x_N]^T \in \Omega$. Ω denotes the decision space. The objective vector $f(x)$ consists of M objective functions, $f_i(x) \in R^M$, i=1, …, M. R^M denotes the objective space.

For an MOPs, a group of non-dominated optimal solutions is called the Pareto optimal set (POS), and the corresponding objective vectors are called the Pareto optimal front (POF). Moreover, the MMOPs have more than one equivalent set of Pareto optimal solutions or at least more than one local Pareto optimal solution for any point on the Pareto front [4]. Therefore, it is quite different between MMOEAs from ordinary MOEAs. The MMOEAs need to satisfy three conditions simultaneously, (1) well-converged, (2) well-distributed in the objective space, and (3) well-distributed in the decision space.

In recent years, numerous MMOEAs have been designed for solving MMOPs. Some of them enhance the diversity of the decision space to find multimodal optimal solutions [5]-[10]. In contrast, some MMOEAs are based on decomposition techniques and integrate niching techniques in the decision space for searching more groups of Pareto optimal solutions [11]-[13]. Moreover, the performance indicators, such as hypervolume indicator, are also adopted in some MMOEAs to guide the evolutionary process [14]-[16].

In this paper, a novel multi-population evolution strategy algorithm is proposed for solving MMOPs, namely MP-MMOES. The original population should be divided into two groups of subpopulations with equal size. The first subpopulation is designed to search for the optimal solutions in objective space that would satisfy conditions (1) and (2). At the same time, the second subpopulation focus to obtain high-quality optimal solutions in the decision space, that need satisfy conditions (1) and (3). For this reason, the multi-population strategy can meet three conditions simultaneously, and improve the decision and objective distributions effectively.

The performance of the proposed algorithm is evaluated on twenty-three competition MMOP test instances in IEEE CEC 2019 [17]. The proposed algorithm can address the MMOPs effectively, and achieve high-quality multiple groups of optimal solutions efficiently. The experimental results indicate

978-1-6654-4195-7/21/$31.00 ©2021 IEEE
DOI 10.1109/ICoIAS53694.2021.00043

the proposed algorithm performs better than five competing state-of-the-art MMOEAs, including Omni-optimizer [5], MO_Ring_PSO_SCD [10], MOEA/D-AD [12], NIMMO [16] and TriMOEA-TA&R [9].

II. Related Works

For the past few years, many MMOEAs have been designed for solving MMOPs with different strategies, including Pareto-based approaches, decomposition-based approaches, and indicator-based approaches.

A. Pareto-based MMOEAs

The Pareto-based MMOEAs prefer the converged solutions with the Pareto dominance principle, and enhance the diversity of the decision space to find multimodal optimal solutions. In [5], the Omni-optimizer is presented to search for multiple equivalent Pareto optimal solutions by using crowding distances in both the decision and objective spaces. In [6], DNEA is proposed to maintain good distribution with sharing functions. In [7], DN-NSGA-II is designed to replace the crowding distance in the objective space with decision space. In [8], SPEA2+ adopt two archives to improve the performance of SPEA2, and the archives keep updating with density qualities.

In [9], TriMOEA-TA&R is presented for multimodal multi-objective optimization, which adopts two-archive and recombination strategies. In [10], MO-Ring-PSO-SCD is designed to search for multiple Pareto subregions that integrated special crowding distance into multi-objective PSO algorithm.

B. Decomposition-based MMOEAs

The Decomposition-based MMOEAs usually decompose an MMOP into a number of single-objective optimization sub-problems, and uses a search heuristic to optimize these sub-problems simultaneously and cooperatively, such as MOEA/D [11]. In [12], MOEA/D-AD is proposed to integrate Addition and Deletion operators with dynamic population size. In [13], a decision space diversity maintenance mechanism is incorporated into MOEA/D for solving MMOPs.

C. Indicator-based MMOEAs

The indicator-based MMOEAs choose some performance indicators to guide the evolutionary process. In [14], the hypervolume indicator and Solow-Polasky diversity technique are adopted for solving MMOPs. In [15], the hypervolume indicator is also integrated to optimize the decision diversity. In [16], a novel niching indicator-based algorithm NIMMO is proposed to solve MMOPs.

III. Proposed Algorithm

In this paper, we present a multi-population based evolution strategy for solving MMOPs, named MP-MMOES. In the proposed algorithm, the whole population would be divided into two subpopulations. The first subpopulation is focused on searching for well-diversified optimal solutions in the objective space. The second subpopulation is designed to maintain diversity in the decision space. The multi-population technique is helpful to improve the objective and decision distributions simultaneously.

Let $P_t^{(i)}$ be the ith individual in population P_t, which is divided into two subpopulations S^{obj} and S^{dec} randomly, as shown in Fig.1.

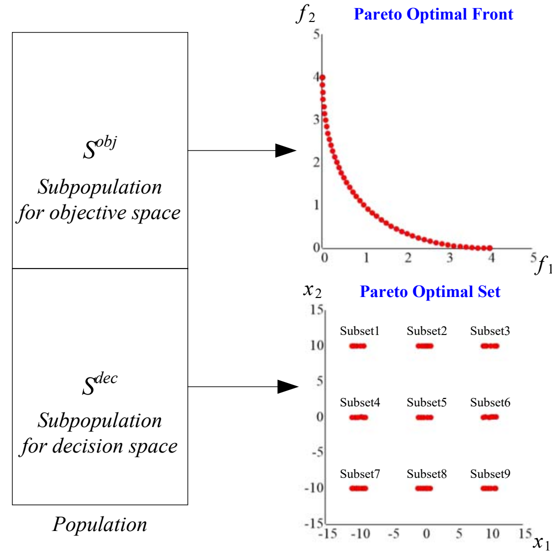

Fig. 1. The proposed algorithm adopts multi-population strategy to search for objective space and decision space independently.

For each generation, the proposed algorithm uses the Polynomial mutation to generate new candidate individuals. Let $NewP_t^{(i)}$ be the newly mutated candidate solution of individual $P_t^{(i)}$. If $NewP_t^{(i)}$ dominates $P_t^{(i)}$, it would replace the $P_t^{(i)}$ since $NewP_t^{(i)}$ has better convergent quality than $P_t^{(i)}$.

Let $DomCount(P_t^{(i)})$ be the function, which calculate the summation of other individuals that dominate $P_t^{(i)}$, as shown in Algorithm 1.

Algorithm 1 *DomCount* ($P_t^{(i)}$)

Input: $P_t^{(i)}$
Output: $Count$

```
Count = 0
for k = 1 to P
    if k = i then continue;
    if (P_t^(k) ≺ P_t^(i)) then
        Count = Count + 1
    end if
end for
return Count
```

If $NewP_t^{(i)}$ and $P_t^{(i)}$ are non-dominated with respect to each other, the values of their $BeDomCount()$ would be compared. When the $BeDomCount()$ of $NewP_t^{(i)}$ is smaller than that of $P_t^{(i)}$, the $NewP_t^{(i)}$ would replace the $P_t^{(i)}$ for less individuals can dominate $NewP_t^{(i)}$. When the value of $BeDomCount\ (NewP_t^{(i)})$ is equal to the value of $BeDomCount(P_t^{(i)})$, the values of MED or $MEDx$ would then be compared.

If the $P_t^{(i)} \in S^{obj}$, the MED [18]-[19] values $MED(P_t^{(i)})$ and $MED(NewP_t^{(i)})$ are calculated. If $MED(NewP_t^{(i)})$ is greater than $MED(P_t^{(i)})$, the $NewP_t^{(i)}$ would replace $P_t^{(i)}$ which implies the $NewP_t^{(i)}$ have better diversity quality than $P_t^{(i)}$.

$$MED\left(P_t^{(i)}\right) = TotalDist\left(P_t^{(i)}\right) \times NearDist\left(P_t^{(i)}\right) \quad (2)$$

where

$$TotalDist\left(P_t^{(i)}\right) = \sum_{j=1}^{P} \sum_{m=1}^{M} |f_n^{(i)} - f_n^{(j)}|.$$

$$NearDist\left(P_t^{(i)}\right) = \min_{j, j \neq i} \sum_{m=1}^{M} |f_n^{(i)} - f_n^{(j)}|.$$

On the other hand, if $P_t^{(i)} \in S^{dec}$, the MEDx [20] values $MEDx(P_t^{(i)})$ and $MEDx(NewP_t^{(i)})$ are calculated. If $MEDx(NewP_t^{(i)})$ is greater than $MEDx(P_t^{(i)})$, the $NewP_t^{(i)}$ would replace $P_t^{(i)}$ which implies the mutated new candidate solution $NewP_t^{(i)}$ is superior to the original $P_t^{(i)}$ in decision space.

$$MEDx\left(P_t^{(i)}\right) = TotalDistX\left(P_t^{(i)}\right) \times NearDistX\left(P_t^{(i)}\right) \quad (3)$$

where

$$TotalDistX\left(P_t^{(i)}\right) = \sum_{j=1}^{P} \sum_{n=1}^{N} |x_n^{(i)} - x_n^{(j)}|.$$

$$NearDistX\left(P_t^{(i)}\right) = \min_{j, j \neq i} \sum_{n=1}^{N} |x_n^{(i)} - x_n^{(j)}|.$$

The computational complexity of the *MEDx* is *O*(*NP*), where *N* is the number of decision variables and *P* is the population size. The pseudocode of the *MEDx* is given in Algorithm 2.

Algorithm 2 *MP-MMOES* Algorithm

Initialization P_t, t =0
for i= 1 to P
    if i mod 2 =0 then $S^{obj} = S^{obj} \cup P_t^{(i)}$
    else $S^{dec} = S^{dec} \cup P_t^{(i)}$
end for
while (t < maximum generation) {
    for i = 1 to P {
        $NewP_t^{(i)} = PolynomialMutation(P_t^{(i)})$
        Objective Functions Calculation $(NewP_t^{(i)})$
        if $(NewP_t^{(i)} \prec P_t^{(i)})$ {
            $P_t^{(i)} = NewP_t^{(i)}$
        else if $(NewP_t^{(i)} \nprec P_t^{(i)})$ and $(P_t^{(i)} \nprec NewP_t^{(i)})$ {
            if $\boldsymbol{DomCount}\left(NewP_t^{(i)}\right) < \boldsymbol{DomCount}\left(P_t^{(i)}\right)${
                $P_t^{(i)} = NewP_t^{(i)}$
            else if $\boldsymbol{DomCount}(NewP_t^{(i)}) = \boldsymbol{DomCount}(P_t^{(i)})$ {
                if $(P_t^{(i)} \in S^{obj}$ and $\frac{\boldsymbol{MED}(NewP_t^{(i)})}{\boldsymbol{MED}(P_t^{(i)})} > 1)$ **or**
                $(P_t^{(i)} \in S^{dec}$ and $\frac{\boldsymbol{MED}_x(NewP_t^{(i)})}{\boldsymbol{MED}_x(P_t^{(i)})} > 1)$ {
                    $P_t^{(i)} = NewP_t^{(i)}$
                } //end if
            } //end if
        } //end if
    } //end for
} //end while

IV. Experimental Results

The proposed MP-MMOES is evaluated on twenty-three benchmark instances on IEEE CEC 2019 competition MMOPs [17], and the performances are compared with five state-of-the-art MMOEAs, including MO_Ring_PSO_SCD [10], Omni-optimizer [5], MOEA/D-AD [12], NIMMO [16], and TriMOEA-TA&R [9]. In our experiment, the population size, *P*, is set to 200, while the maximum iteration is set to 300. The Polynomial mutation distribution index [21] is set to 20, In order to make a fair comparison, all the competing algorithms are run 30 times independently for each MMOP.

In the comparison, two indicators IGD [22] and IGDx [23] are adopted to evaluate the performance of these competing algorithms. The smaller IGD or IGDx value means the obtained solutions have better diversity and convergence in the objective space or in the decision space respectively.

$$IGD = \frac{(\sum_{i=1}^{|PF|} Dist_i^2)^{1/2}}{|PF|}, \quad (4)$$

$$IGDx = \frac{(\sum_{j=1}^{|PS|} Dist_j^2)^{1/2}}{|PS|}, \quad (5)$$

where $Dist_i$ of the IGD metric denotes the Euclidean distance between *i*th sampled reference objective vector and obtained nearest objective vector. Moreover, $Dist_j$ of the IGDx metric denotes the Euclidean distance between the *j*th reference decision vector and obtained nearest decision variables.

Table I lists the IGD values obtained using the six competing MMOEAs. The proposed MP-MMOES achieved significantly better IGD performance for sixteen MMOPs, which wins fifteen instances. Omni-optimizer also performed best on seven instances. NIMMO yielded the best results on one test instance. For Omni test instance, MMF2, MMF3, MMF4, MMF6, and MMF8 instances, MP-MMOES achieve better performance than other competing MMOEAs, except Omni-optimizer. For MMF9, NIMMO achieves the best performance, while MP-MMOES and Omni-optimizer are not far behind.

Table II lists the IGDx values obtained by the competing MMOEAs. Compared with the other competing MMOEAs, MP-MMOES achieved significantly better IGDx performance for thirteen MMOPs. For the remaining MMOPs, NIMMO performed best on four instances, MOEA/D-AD and TriMOEA-TA&R also win three instances, respectively. Although Omni-optimizer obtained the best IGDx values on SYM Part2, SYM Part3, and MMF4 instances, MP-MMOES also performed better than other MMOEAs. For MMF10, MMF11, and MMF12 instances, the proposed MP-MMOES achieved good IGDx values, just a little worse than NIMMO.

As shown in Table I and Table II, MP-MMOES achieves the best performance on most of the IEEE CEC2019 MMOPs in both IGD and IGDx metrics. The experimental results prove that MP-MMOES can handle the MMOPs with various characterizes, including convex, concave, disconnected, and sphere Pareto fronts. In addition, these MMOP instances have 2 to 27 Pareto subregions. The experimental results also demonstrate the ability to find multiple sets of well-distributed and well-converged optimal solutions in the decision space.

TABLE I. AVERAGE IGD VALUES OVER 30 RUNS ON BENCHMARK INSTANCES (POPULATION SIZE 200), WHERE THE BEST MEAN IS SHOWN IN A GRAY BACKGROUND

Problem	MP-MMOES	MO_Ring_PSO_SCD	Omni-optimizer	MOEA/D-AD	NIMMO	TriMOEA-TA&R
OMNI TEST	0.01251123	0.029150124(+)	0.008958462(–)	0.012637087(=)	0.024598026(+)	0.015126827(+)
SYM PART1	0.00897592	0.021807789(+)	0.009521222(+)	0.017766730(+)	0.034295248(+)	0.040351677(+)
SYM PART2	0.01004566	0.022575431(+)	0.011739039(+)	0.017760622(+)	0.035765847(+)	0.030371377(+)
SYM PART3	0.01116590	0.023256683(+)	0.012636299(+)	0.016262678(+)	0.048581612(+)	0.026949094(+)
MMF1	0.00276004	0.005121772(+)	0.002815454(+)	0.006502579(+)	0.005040815(+)	0.003179426(+)
MMF2	0.00537555	0.005361835(=)	0.003353320(–)	0.013881331(+)	0.011629751(+)	0.007202187(+)
MMF3	0.00504986	0.005290680(+)	0.003214919(–)	0.017465270(+)	0.009359596(+)	0.005974700(+)
MMF4	0.00272586	0.004610905(+)	0.002617184(–)	0.003136242(+)	0.005434900(+)	0.003330633(+)
MMF5	0.00278193	0.005059484(+)	0.002819489(+)	0.005939438(+)	0.003843573(+)	0.003219364(+)
MMF6	0.00275766	0.004336623(+)	0.002673216(–)	0.009208363(+)	0.004303962(+)	0.003129046(+)
MMF7	0.00269708	0.004664684(+)	0.002812777(+)	0.003251445(+)	0.002749795(+)	0.003421886(+)
MMF8	0.00351207	0.005222054(+)	0.003021578(–)	0.006653161(+)	0.013075597(+)	0.003671090(+)
MMF9	0.01124202	0.032835218(+)	0.011532801(+)	0.014723816(+)	0.009595338(–)	0.072431150(+)
MMF10	0.05685035	0.201809301(+)	0.196936485(+)	0.201469069(+)	0.203597691(+)	0.232581952(+)
MMF11	0.07961368	0.111134791(+)	0.093753732(+)	0.096695782(+)	0.087955047(+)	0.168825871(+)
MMF12	0.04027870	0.055809815(+)	0.082851825(+)	0.084465142(+)	0.091129024(+)	0.085663818(+)
MMF13	0.07036060	0.140756921(+)	0.129059300(+)	0.151105279(+)	0.155633659(+)	0.243840159(+)
MMF14	0.08174011	0.106392551(+)	0.087486389(+)	0.083680814(+)	0.108886753(+)	0.089755198(+)
MMF14A	0.08460216	0.103348602(+)	0.089508155(+)	0.088885890(+)	0.126412248(+)	0.095896802(+)
MMF15	0.17141854	0.191021894(+)	0.198716264(+)	0.185994005(+)	0.203966567(+)	0.209279524(+)
MMF15A	0.17427857	0.186060373(+)	0.192931611(+)	0.192538464(+)	0.223188539(+)	0.200169061(+)
MMF1Z	0.00260765	0.004394332(+)	0.002626091(+)	0.006512066(+)	0.004145604(+)	0.003240226(+)
MMF1E	0.00632096	0.004837187(–)	0.003008280(–)	0.074925971(+)	0.004988975(–)	0.003952197(–)

Wilcoxon's rank sum test at a 0.05 significance level is performed between MP-MMOES and compared MMOEAs. "+" means MP-MMOES better than compared algorithm, "–" means compared algorithm better than MP-MMOES, "=" means not comparable)

TABLE II. AVERAGE IGDX VALUES OVER 30 RUNS ON BENCHMARK INSTANCES (POPULATION SIZE 200), WHERE THE BEST MEAN IS SHOWN IN A GRAY BACKGROUND

Problem	MP-MMOES	MO_Ring_PSO_SCD	Omni-optimizer	MOEA/D-AD	NIMMO	TriMOEA-TA&R
Omni Test	0.02632202	0.217196057(+)	0.131158507(+)	0.081893471(+)	0.217575998(+)	0.594181794(+)
SYM Part1	0.05932537	0.096149204(+)	1.173611504(+)	0.041591252(–)	0.048317548(–)	0.023229720(–)
SYM Part2	0.06704590	0.097445041(+)	0.097448507(+)	0.048892647(+)	0.063278998(–)	4.155618496(+)
SYM Part3	0.05121727	0.851996784(+)	3.050050628(+)	0.034863555(+)	0.081800845(+)	1.620846507(+)
MMF1	0.03762181	0.063353978(+)	0.045215521(+)	0.071513259(+)	0.053231768(+)	0.049364585(+)
MMF2	0.00741552	0.009132735(+)	0.031248666(+)	0.028723763(+)	0.052812145(+)	0.067378944(+)
MMF3	0.00727242	0.011067556(+)	0.027324559(+)	0.031608785(+)	0.029581061(+)	0.020799817(+)
MMF4	0.02049264	0.036325339(+)	0.030923922(+)	0.017905388(+)	0.042916214(+)	0.024458536(+)
MMF5	0.06496427	0.102276120(+)	0.081714047(+)	0.080449518(+)	0.076950091(+)	0.085264364(+)
MMF6	0.05597783	0.079549655(+)	0.080433137(+)	0.123508902(+)	0.094966913(+)	0.069074403(+)
MMF7	0.01977541	0.031406961(+)	0.022861407(+)	0.027485326(+)	0.017435345(–)	0.018530028(–)
MMF8	0.04974763	0.082158474(+)	0.096914560(+)	0.109575408(+)	0.172983742(+)	0.573401878(+)
MMF9	0.00683108	0.010021181(+)	0.014822142(+)	0.028384815(+)	0.003361935(–)	0.003214856(–)
MMF10	0.02695622	0.151342778(+)	0.200798997(+)	0.162241019(+)	0.005874503(–)	0.201518847(+)
MMF11	0.18725754	0.182463070(–)	0.249403697(+)	0.252730929(+)	0.005178888(–)	0.252469858(+)
MMF12	0.04264283	0.144709677(+)	0.245224634(+)	0.057814483(+)	0.003454885(–)	0.248161186(+)
MMF13	0.08438545	0.207209987(+)	0.256424454(+)	0.26776513(+)	0.152421393(+)	0.262319177(+)
MMF14	0.05391435	0.070215794(+)	0.056658273(+)	0.049692623(–)	0.069142571(+)	0.037778377(–)
MMF14a	0.06657818	0.081579614(+)	0.077396503(+)	0.080790013(+)	0.117392033(+)	0.070475720(+)
MMF15	0.15399948	0.165294286(+)	0.265308491(+)	0.235141253(+)	0.173706488(+)	0.271423071(+)
MMF15A	0.15002052	0.174037831(+)	0.216245775(+)	0.210341619(+)	0.217304018(+)	0.223909506(+)
MMF1z	0.02695412	0.041882876(+)	0.032226072(+)	0.050524589(+)	0.034615409(+)	0.052162199(+)
MMF1e	0.21090805	0.309876589(+)	2.054671704(+)	0.646411251(+)	0.867156463(+)	1.818341024(+)

Wilcoxon's rank sum test at a 0.05 significance level is performed between MP-MMOES and compared MMOEAs. "+" means MP-MMOES better than compared algorithm, "–" means compared algorithm better than MP-MMOES, "=" means not comparable)

V. CONCLUSION

In this paper, we propose a multi-population based evolutionary algorithm to solve MMOPs. The whole population is divided into two subpopulations in our algorithm. The first subpopulation is designed to find well-converged and well-distributed non-dominated optimal solutions in objective space. The second subpopulation is focused on searching for multiple groups of multimodal optimal solutions in decision space. In order to obtain good distributed optimal solutions, the MED and MEDx methods are adopted to maintain diversity in the objective space and the decision space, respectively. The performance of MP-MMOES is compared against five state-of-the-art MMOEAs, including MO_Ring_PSO_SCD, Omni-optimizer, MOEA/D-AD, NIMMO, and TriMOEA-TA&R. The experimental results demonstrate the proposed MP-MMOES provides competing performance than compared MMOEAs on twenty-three IEEE MMOPs in IGD, IGDx metrics.

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China (Grant Nos. 61702383, U1803262, 61602350).

REFERENCES

[1] Y. Wang, H.X. Li, G.G. Yen and W. Song, "MOMMOP: Multiobjective optimization for locating multiple optimal solutions of multimodal optimization problems," IEEE Trans. Cybern., Vol. 45. No. 4, April 2015, pp. 830-843.

[2] K.M. Woldemariam and G.G. Yen, "Vaccine enhanced artificial immune system for multimodal function optimization," IEEE Transactions on Systems, Man and Cybernetics, Part B: Cybernetics, Vol. 40, No.1, February 2010, pp. 218-228.

[3] K. Zhang, C. N. Shen, J. J. He, and G. G. Yen, "Knee based multimodal multi-objective evolutionary algorithm for decision making," Information Sciences, Vol. 544, January 2021, pp. 39-55.

[4] Y. Liu, H. Ishibuchi, G. G. Yen, Y. Nojima and N. Masuyama, "Handling imbalance between convergence and diversity in the decision space in evolutionary multimodal multiobjective optimization," IEEE Trans. Evol. Comput., vol. 24, no. 3, pp. 551–565, 2020.

[5] K. Deb and S. Tiwari, "Omni-optimizer: A procedure for single and multiobjective optimization", Proc. 3rd Int. Conf. Evol. Multicriterion Optimization (EMO '05), vol. 3410, pp. 41-65, 2005.

[6] Y. Liu, H. Ishibuchi, Y. Nojima, N. Masuyama, and K. Shang, "A double-niched evolutionary algorithm and its behavior on polygon-based problems," in Proc. PPSN, pp. 262–273, Springer, 2018.

[7] J. Liang, C. Yue, and B. Qu, "Multimodal multi-objective optimization: a preliminary study," in Proceedings of the 2016 IEEE Evolution Computation Congres, pp. 2454–2461, 2016.

[8] M. Kim, T. Hiroyasu, M. Miki, and S. Watanabe, "SPEA2+: Improving the performance of the strength Pareto evolutionary algorithm 2," in Proc. PPSN, 2004, pp. 742–751.

[9] Y. Liu, G. G. Yen, and D. Gong, "A multi-modal multi-objective evolutionary algorithm using two-archive and recombination strategies," IEEE Trans. Evol. Comput., vol. 23, no. 4, pp. 660–674, 2019.

[10] C. Yue, B. Qu, and J. Liang, "A multi-objective particle swarm optimizer using ring topology for solving multimodal multi-objective problems," IEEE Trans. Evol. Comput., vol. 22 no. 5, pp. 805-817, Oct. 2018.

[11] Q. Zhang and H. Li, "MOEA/D: A multiobjective evolutionary algorithm based on decomposition," IEEE Trans. Evol. Comput., vol. 11, no. 6, pp. 712-731, 2007.

[12] C. Hu and H. Ishibuchi, "Incorporation of a decision space diversity maintenance mechanism into MOEA/D for multi-modal multi-objective optimization," in Proc. GECCO, 2018, pp. 1898-1901.

[13] R. Tanabe and H. Ishibuchi, "A Decomposition-Based Evolutionary Algorithm for Multi-modal Multi-objective Optimization," in Proc. PPSN, 2018, pp. 249-261.

[14] T. Ulrich, J. Bader, and L. Thiele, "Defining and Optimizing Indicator-Based Diversity Measures in Multiobjective Search," in Proc. PPSN, 2010, pp. 707-717.

[15] H. Ishibuchi, M. Yamane, N. Akedo, and Y. Nojima, "Two-objective solution set optimization to maximize hypervolume and decision space diversity in multiobjective optimization," in Proc. SCIS, 2012, pp. 1871-1876.

[16] R. Tanabe and H. Ishibuchi, "A niching indicator-based multi-modal many-objective optimizer," Swarm and Evolutionary Computation, 2019, pp. vol. 49, 134–146.

[17] J. J. Liang, B. Y. Qu, D. W. Gong, C. T. Yue, "Problem definitions and evaluation criteria for the CEC 2019 special session on multimodal multiobjective optimization," in Proc. CEC, 2019, Technical Report 201912.

[18] K. Zhang, Z.W. Xu, S.L. Xie and G.G. Yen, "Evolutionary strategy based many objective evolutionary algorithm through vector equilibrium,", IEEE Trans. Cybern., 2019, DOI:10.1109/TCYB.2019.2960039.

[19] K. Zhang, C.N. Shen, X.M. Liu, and G.G. Yen, "Multi-objective evolutionary strategy for dynamic multi-objective optimization," IEEE Trans. Evol. Comput., vol. 24, no. 5, pp. 974 - 988, 2020.

[20] Kai Zhang, Minshi Chen, Xin Xu, Gary G.Yen, "Multi-objective evolution strategy for multimodal multi-objective optimization," Applied Soft Computing, vol. 101, 107004, 2021

[21] K. Deb and S. Tiwari, "Omni-optimizer: A generic evolutionary algorithm for single and multi-objective optimization," European Journal of Operational Research, vol. 185, no. 3, pp. 1062-1087, 2008

[22] E. Zitzler, L. Thiele, M. Laumanns, C. M. Fonseca, and V. G. da Fonseca, "Performance assessment of multiobjective optimizers: an analysis and review," IEEE Trans. Evol. Comput., vol 7, no. 2, 117–132, 2003.

[23] A. Zhou, Q. Zhang, and Y. Jin, "Approximating the set of Pareto-optimal solutions in both the decision and objective spaces by an estimation of distribution algorithm," IEEE Trans. Evol. Comput., vol. 13, no. 5, pp. 1167-1189, Oct. 2009.