Multi-Objective Optimization for Multimodal Multi-Objective Multi-Point Shortest Path Problem Considering Unforeseeable Road Eventualities

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***Abstract*— Multi-objective multi-point shortest path planning problems are commonly encountered in real-world applications. Numerous path planning algorithms have been proposed to accommodate different model assumptions. However, most existing algorithms can only identify a subset of the Pareto optimal paths and overlook equivalent Pareto optimal paths. Relying solely on a subset of Pareto optimal solutions is insufficient to effectively respond to unforeseeable road eventualities in the real-world traffic environment. In this paper, multi-objective multi-point shortest path planning problem is modeled as a multimodal multi-objective optimization problem with necessary points constrains. A multimodal multi-objective evolutionary algorithm using constraint dominance principle-based path comparison strategy and path similarity-based multimodal solutions selection strategy is proposed to address this problem. The proposed constraint dominance principle-based path comparison strategy can effectively navigate through large infeasible regions by relaxing necessary point constraints, thereby obtaining a true constrained Pareto front. The proposed path similarity-based multimodal solutions selection strategy can effectively balance the distribution of solutions in the decision space, thereby preserving multiple equivalent optimal solutions. The proposed algorithm is compared with five state-of-the-art path planning algorithms from the benchmark test suite derived from the 2021 IEEE CEC path planning competition, where city maps are adapted from real transportation networks in Chinese cities, in our experiments. The exceptional performance is demonstrated through thirty independent runs, yielding experimental results that showcase the superiority of the proposed algorithm on the test problem set. This superior performance highlights the potential for designing more resilient path planners suitable for scenarios affected by unpredictable road eventualities.**

***Index Terms*— Multi-objective shortest path planning, constrained multi-objective optimization, multimodal multi-objective optimization, multi-objective evolutionary algorithm.**

I. Introduction

MULTI-OBJECTIVE multi-point shortest path problem aims to find a set of Pareto optimal paths to reach a specified goal from a fixed start via several necessary points and balance all objective functions, which are always conflicting. In recent years, the multi-objective multi-point shortest path problem has been extensively studied in logistics science and transportation, examples of this type of problem include multi-objective vehicle routing problems [1], multi-objective travelling salesman problems [2], and multi-objective tourist path planning problem [3]. Various path planning algorithms have been proposed for different model assumptions [4]. Most existing algorithms ignore the equivalent optimal paths with the same objective function values and can only find part of optimal paths. However, in many practical applications, such as special operations, disaster rescues, and emergency responses, it is necessary to obtain the optimal path plans as many as possible to exclude the impact of temporary or unavoidable factors such as traffic accidents, temporary diversions, road construction, and road closures caused by extreme harsh environments [5], [6], [7].

In applied mathematics, when the optimal solution in the objective space corresponds to multiple different optimal decision vectors in the decision space, such problems are called multimodal optimization problems [8], for example, industrial design optimization problems [9], production scheduling problems [10], feature selection problems [11], data mining problems [12]. Fig. 1 shows a toy example of a multimodal multi-objective multi-point shortest path planning (MMMSPP) problem. The path length and the length of passing congestion areas are the two objectives considered simultaneously. The blue point represents the start, the green point signifies the goal, the yellow point denotes the necessary point and the red points indicate areas of congestion. In Fig. 1, there are two

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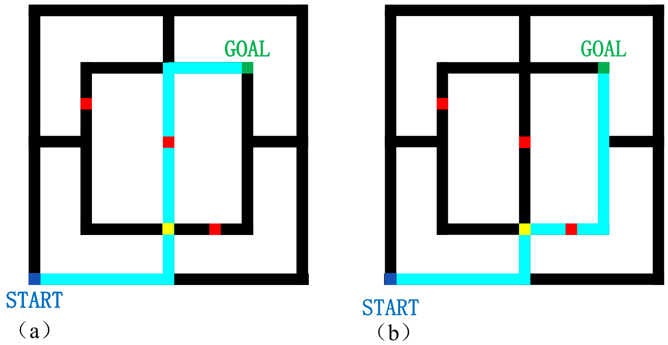


Fig. 1. Two equivalent optimal shortest paths, of the same length, traversing necessary points and with the same length of congested areas.

optimal paths that pass through the necessary point, have the same path lengths and pass through the same length of congestion areas. Failure to identify diverse shortest paths may prevent decision-makers from considering solutions that satisfy their preferences. It is worth mentioning that these two paths are equally important, and they pass through different intermediate points, which can bring diverse choices to decision-makers with multiple preferences. Once one path becomes unavailable due to unavoidable factors, decision-makers can easily and quickly switch to another path.

The difficulty of the traditional multi-objective shortest path problems is that as the number of objectives increases, the dominant relationship between solutions becomes difficult to compare, and the number of non-dominated solutions increases exponentially, which increases the difficulty of the search. Commonly used methods to solve multi-objective shortest path problems include multi-objective A* algorithms [13], [14], multi-objective Dijkstra algorithms [15], [16], and multi-objective evolutionary algorithms (MOEAs) [17], [18]. On one hand, these algorithms focus on developing pruning techniques to eliminate infeasible paths at the earliest stages of the search process. On the other hand, they aim to improve algorithmic search performance by enhancing solution generation mechanisms to obtain more explicit lower bounds.

However, the existing algorithms fail to effectively address the MMMSPP problem due to the challenges posed by its multimodal and multi-point nature. From the multi-point aspect, most methods consider all necessary points as mandatory conditions, giving rise to the following challenges. The first challenge lies in the diversity aspect. The presence of a large and infeasible region poses difficulties in exploring certain fronts. Secondly, there is a feasibility challenge. Either an extremely small feasible region hinders effective search or an excessively large infeasible region obstructs the search direction, leading to local optima. Finally, there is a convergence challenge. Constraints impede algorithmic convergence, making it arduous to achieve the minimization objective during the evolutionary process. From the multimodal aspect, most existing methods can only identify one Pareto front (PF) and lack the capability to preserve Pareto optimal solution set (PS) with multiple equivalent optimal solutions. This limitation stems from two main factors. Firstly, the absence of an effective search mechanism leads to algorithm convergence on a partial, resulting in the omission of optimal solutions. Secondly, insufficient consideration is given to the distribution of solutions in the decision space. This leads to missed opportunities for potential solutions with limited dispersion in the objective space but significant separation in the decision space.

To effectively address the MMMSPP problems, this paper proposes a MOEA namely MMOEA-CDP for path search in the context of multimodal multi-objective multi-point path planning. To address the challenges posed by multi-point aspect, a constraint dominance principle (CDP) based path comparison strategy is proposed. Each necessary point is treated as a separate constraint, and the extent to which the candidate solution satisfies these constraints is considered as a criterion for assessing the quality of the solution. By relaxing the constraints, the algorithm can pass through a large area of infeasibility and reach the true constrained Pareto front (CPF). To effectively tackle the complexities inherent in multimodal aspect, a multimodal solution selection strategy grounded in path similarity is proposed. Abandoning the approach of preserving diversity in objective space employed by most algorithms, this strategy maintains the diversity of candidate solutions within the decision space through a selection method grounded in path similarity. This enables the algorithm to retain multiple equivalent optimal solutions.

Given the above, the main contributions can be encapsulated as follows.

- The multi-objective multi-point shortest path planning problem is formulated as a constrained multimodal optimization problem. In the proposed problem, the necessary points are represented as constraints. The goal of the problem is to find all equivalent Pareto optimal solutions that minimize each objective and pass through all necessary points.
- A CDP-based path comparison strategy is proposed which regards the number of necessary points as a criterion for evaluating the quality of a candidate path, thereby relaxing the constraints.
- A path similarity-based multimodal solution selection strategy is proposed that identifies solutions with superior diversity within the decision space to preserve multiple Pareto optimal solutions.

The subsequent sections of this paper are structured as follows: Section II provides a concise overview of the relevant research literature. Section III presents the theoretical problem statement, while Section IV elaborates on the proposed algorithm. The experimental setup is explained in Section V. In Section VI, we present the experimental results obtained from our study, and finally, in Section VII, we provide a comprehensive conclusion.

II. Related Work

In this section, we will initially present some existing research on multi-objective shortest path planning in Section II-A. Subsequently, we will review the current literature on multimodal multi-objective optimization in Section II-B. Finally, we give the motivation of this work in Section II-C.

A. Multi-Objective Shortest Path Planning

The algorithms for solving multi-objective multi-point shortest path planning problems can be classified into two categories, exact algorithms and heuristic algorithms. The classical exact algorithms encompass the multi-objective Dijkstra algorithm and the multi-objective A* algorithm. Hansen [19] initially extended the Dijkstra algorithm [20] to address the bi-objective shortest path problem, and Martins [21] further expanded upon this approach for the multi-objective scenario. When the problem specifies the goal, the A* algorithm [22] can estimate the proximity between a given node and the goal through heuristic information to improve the search efficiency. Stewart and White [23] outlined the first multi-objective extension of A* namely MOA*. Based on these results, an increasing number of multi-objective A* algorithms have been proposed. The categorization of multi-objective A* algorithms can be classified into two distinct approaches: node expansion and label expansion. Node expansion-based algorithms extend the node expansion policy of classic MOA* to different contexts, like algorithms MOA** for search with non-consistent lower bounds [24], BCA* for compromise solutions [25], or METAL-A* for goal-based preferences [26]. The classic label expansion-based algorithm is NAMOA* [27]. Recent attempts to improve this algorithm include parallel search [28] and the dimensionality reduction technique [29].

However, the time complexity of the exact algorithm increases significantly for solving large-scale multi-objective shortest path planning problems, leading to a continuous decline in its performance. Consequently, heuristic algorithms have garnered increasing attention [58], [59], [60], [61]. When it comes to multi-objective path planning, classical heuristic algorithms commonly employed for addressing this category of problems primarily include genetic algorithms, ant colony optimization, variable neighborhood search, and greedy randomized adaptive search [62], [63], [64], [65]. Genetic algorithm-based heuristics enhance the classic NSGA-II [30] by denoising autoencoder [31], clustering [32], and local search [33]. Ant colony algorithms [34] simulate the pathfinding behavior of ants and selects the optimal path according to the pheromone concentration. Recent improvement attempts include multiple mutation operator [35] and greedy search [36]. Variable neighborhood search [51] and greedy randomized adaptive search [52] are widely used in path planning for uncrewed aerial vehicles and robotics.

B. Multimodal Multi-Objective Optimization

Multimodal optimization refers to optimization problems where there are multiple equally optimal solutions, each corresponding to a distinct decision vector in the decision space. In traditional multi-objective optimization, the primary focus is on finding the PF, which consists of solutions that cannot be improved in any objective without worsening others. However, in multimodal multi-objective optimization, the goal is not only to identify the PF but also to ensure that all the equally optimal solutions are discovered and retained, despite their possible differences in the decision space. A key challenge in multimodal multi-objective optimization is maintaining decision space diversity, as traditional methods like crowding distance or dominance-based selection often prioritize objective space diversity [37]. Consequently, traditional MOEAs may overlook or lose equivalent optimal solutions when addressing multimodal multi-objective problems [11].

In recent years, many multimodal multi-objective evolutionary algorithms (MMEAs) with different mechanisms have been proposed for solving multimodal multi-objective optimization problems [53]. Omni-optimizer [37], one of the most representative MMEAs, introduces an alternative crowding distance to preserve solution diversity in both the objective and decision spaces. DNEA [38] and DN-NSGAII [39] build on Omni-optimizer by incorporating dual niche and decision space-based niching strategies, respectively, to enhance diversity preservation. DNPD [40] integrates decision space information into Pareto dominance, employing dynamic niches to retain well-distributed solutions. APHMA [41] combines hierarchical environmental selection with affinity propagation clustering to eliminate similar solutions in the decision space while preserving diverse PSs. ArchiveUpdateLQ [55] identifies -locally optimal solutions to enable comprehensive exploration of the decision space. BOEA [56] redefines multimodal optimization as a bi-objective problem, explicitly separating convergence and diversity objectives, with hierarchical clustering enhancing diversity preservation. Lastly, ClusteringGA [57] adopts a clustering-based niching method with affinity propagation clustering to identify diverse PSs.

To the best of our knowledge, despite the extensive empirical evidence supporting the effectiveness of the aforementioned MMEAs in addressing multimodal multi-objective problems involving real numbers, there is a paucity of research focusing on discrete problem domains [54]. Real-valued optimization problems can be very different from discrete ones for MOEAs to deal with. MOEAs which work well on real-valued problems can easily get stuck in discrete search space, even in very different places in every execution [50]. The primary reason that MMEAs have rarely been studied in discrete problem domain may lie in the necessity for them to conduct selection operations based on the distance between candidate solutions in the decision space. For the multi-objective shortest path problems, scholars have made the following attempts. In terms of enhancing diversity in the decision space, MMEAs enhance the classic NSGA-II through distinct strategies, such as MMEA-SES [42], which incorporates improved environmental selection, and MACOSX [43], which utilizes a topological map. From the perspective of enhancing search capabilities, MDACO [44] introduces an enhanced ant colony optimization algorithm as the evolutionary operator.

C. Motivation

The MMMSPP exhibits the characteristics of being multi-objective, multi-constraint, and multimodal. From the multi-objective perspective, as the number of objective functions increases, so does the number of non-dominated solutions. However, this also leads to dominated impedance and exponentially increased problem difficulty. The conventional algorithm employed for solving multi-objective shortest

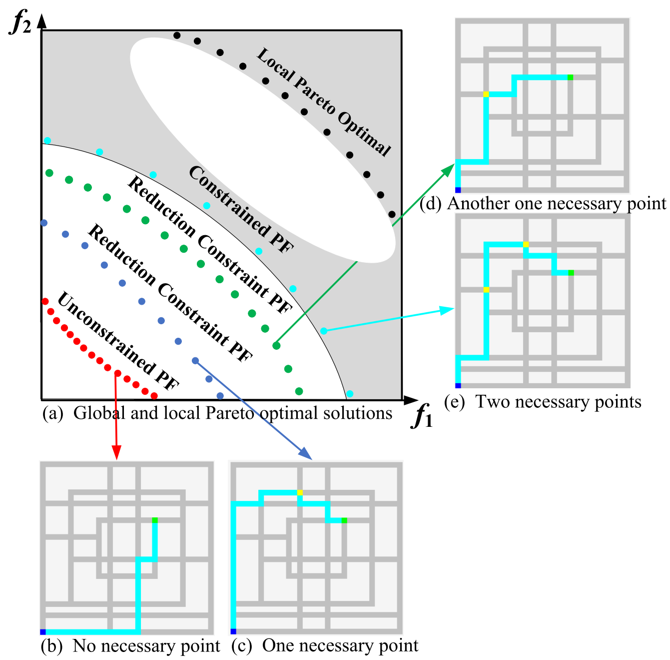


Fig. 2. Illustration of a dual-objective MMMSPP with two necessary points. (a) The objective space of the MMMSPP with a dual-objective and two necessary points. (b)-(e) The corresponding solutions in the objective space.

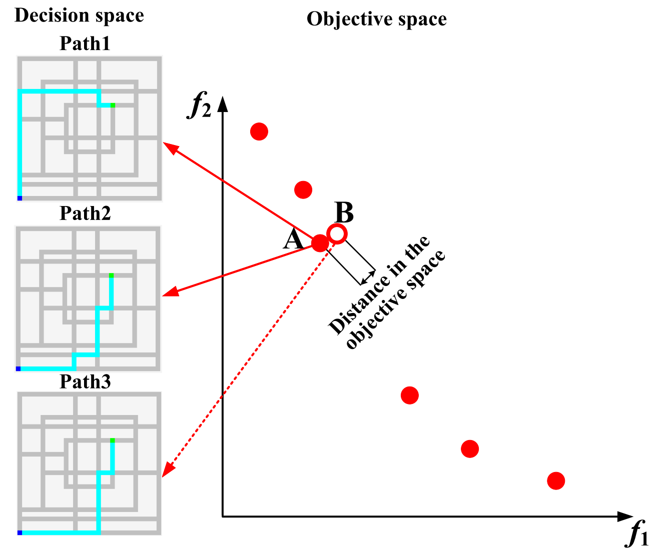


Fig. 3. Illustration of decision space and objective space of a dual-objective MMMSPP. Due to the negligible distance between point **B** and point **A**, point **B** will be eliminated in the objective space, so path3 cannot become path2 by simply mutating.

path planning problems utilizes the objective weighted sum approach. The proposed approach encounters several limitations. Firstly, determining an appropriate weight assignment for conflicting objective functions is challenging. Secondly, a singular model of weight assignment fails to encompass non-continuous PF or those with complex shapes. Lastly, the weight sum approach tends to prioritize solutions within the convex hull and overlooks Pareto optimal solutions in non-convex regions [4].

From the perspective of multi-constraint nature, balancing convergence and constraint satisfaction is the primary difficulty in solving the MMMSPP. The objective space of the MMMSPP with a dual-objective and two necessary points is illustrated in Fig. 2 (a), where the two minimization objectives are the length and the crowding degree of the path. The white region represents the infeasible region caused by the necessary point constraint, and the cyan points depict the true CPF. When the necessary point constraints are eliminated from the problem, the optimal solutions are located on the red unconstrained Pareto front (UPF) as shown in Fig. 2 (b). When the problem is reduced to a single necessary point constraint, the optimal solutions are located on the reduction-constrained PF (RCPF) represented by blue and green colors. Most of the current heuristic algorithms for solving multi-objective shortest path planning problems [31], [32], [33], [34], [35], [36] emphasize convergence and ignore feasibility. The population is easy to converge to the UPF, obtaining feasible solutions that satisfy necessary points constraints remains challenging. Most of the existing exact algorithms [24], [25], [26], [27], [28], [29] tend to prioritize feasibility excessively, hindering the population from traversing a vast infeasible region and converging towards the local optimum. Nevertheless, infeasible solutions possess inherent value. As illustrated in Fig. 2 (c) and (d), solutions that satisfy a single necessary point constraint also hold the potential to transform into solutions on CPF, as depicted in Fig. 2 (e), with minor adjustments. Inspired by this, in this paper, a CDP-based path comparison strategy is proposed to relax the constraint. The number of satisfied necessary points is considered as the constraint satisfaction degree, enabling the population to traverse the extensive infeasible region while retaining infeasible solutions with potential advantages, such as those on RCPF. Both convergence and feasibility are comprehensively addressed.

From the perspective of multimodality, the difficulty of the MMMSPP problem is preserving different paths with the same objective value. Fig. 3 shows the decision space and the objective space of a bi-objective MMMSPP. The decision space perspective reveals that Path1 and Path2 exhibit distinct dissimilarities. However, they share identical objective values and both correspond to point **A** in the objective space. Path3 exhibits numerous overlapping passing points with Path2, and its objective function values correspond to point **B** in the objective space. Assuming that the algorithm has acquired Path1 and Path3, it is observed that only a small amount of transition is required for Path3 to transform into Path2. In the objective space, points **A** and **B** exhibit excessive proximity, leading to the exclusion of point **B** by conventional multi-objective optimization algorithms [30], [31], [32], [33], [34], [35], [36]. However, to retain both Path1 and Path2, it is imperative not to eliminate Path3. Therefore, to ensure the algorithm's capability in preserving multiple equivalent optimal solutions, this paper proposes a multimodal solution selection strategy based on path similarity. The proposed strategy introduces a path similarity index to assess the diversity of individuals within the decision space and selectively retain optimal solutions that exhibit superior diversity.

III. Problem Formulation

Constrained multi-objective optimization is concerned with the optimization of multiple objective criteria simultaneously while satisfying constraints, it can be expressed as follow:

$$\begin{aligned}&\min_{\mathbf{x}\in\Omega} / \max \vec{f}(\overrightarrow{\mathbf{x}})\\ &= \min / \max \left(f_1(\overrightarrow{\mathbf{x}}), f_2(\overrightarrow{\mathbf{x}}), f_3(\overrightarrow{\mathbf{x}}), \ldots, f_M(\overrightarrow{\mathbf{x}})\right)^{\mathrm{T}}\\ &\quad \text{s.t.} \begin{cases} g_s(\overrightarrow{\mathbf{x}}) \le 0, s = 1, \ldots, S \\ h_t(\overrightarrow{\mathbf{x}}) = 0, t = 1, \ldots, T \end{cases}\end{aligned} \tag{1}$$

where $\overrightarrow{\mathbf{x}} = (x_1, x_2, x_3, \ldots, x_D) \in \Omega$ expresses the decision vector with D decision variables x_i, $i = 1, \ldots, D$ within the decision space Ω. $\vec{f}(\overrightarrow{\mathbf{x}}) \in R^M$ is the objective vector with M objective functions to be minimized/maximized and R^M indicates the objective space. In addition, the optimal solutions need to satisfy S inequality constraints and T equality constraints. $g_s(\overrightarrow{\mathbf{x}})$ is the function of the s-th inequality constraint, and $h_t(\overrightarrow{\mathbf{x}})$ is the function of the t-th equality constraint. Due to conflicting objective functions, it is infeasible to seek a singular optimal solution that satisfies all objectives. The following definitions are well-established in the field of multi-objective optimization and are widely utilized for formalizing optimization problems in this domain.

Pareto dominance: Given two solutions $\overrightarrow{\mathbf{x}} = (x_1, x_2, x_3, \ldots, x_D) \in \Omega$ and $\overrightarrow{\mathbf{y}} = (y_1, y_2, y_3, \ldots, y_D) \in \Omega$, $\overrightarrow{\mathbf{x}} \prec \overrightarrow{\mathbf{y}}$, if and only if the following conditions are satisfied.

$$\begin{aligned}&\overrightarrow{\mathbf{x}} \prec \overrightarrow{\mathbf{y}} \text{ iff:}\\ &\quad \forall m \in \{1, \ldots, M\}, \text{ and } \exists j \in \{1, \ldots, M\}\\ &\begin{cases} f_m(\overrightarrow{\mathbf{x}}) \le f_m(\overrightarrow{\mathbf{y}}), f_j(\overrightarrow{\mathbf{x}}) < f_j(\overrightarrow{\mathbf{y}}) \text{ if } \min \vec{f}(\overrightarrow{\mathbf{x}}) \\ f_m(\overrightarrow{\mathbf{x}}) \ge f_m(\overrightarrow{\mathbf{y}}), f_j(\overrightarrow{\mathbf{x}}) > f_j(\overrightarrow{\mathbf{y}}) \text{ if } \max \vec{f}(\overrightarrow{\mathbf{x}}) \end{cases}\end{aligned} \tag{2}$$

Pareto optimal solution: For a solution $\overrightarrow{\mathbf{x}^*}$, if $\nexists \overrightarrow{\mathbf{x}}$ such that $\overrightarrow{\mathbf{x}} \prec \overrightarrow{\mathbf{x}^*}$, then $\overrightarrow{\mathbf{x}^*}$ is termed a Pareto optimal solution.

Equivalent Pareto optimal solutions: For two Pareto optimal solutions $\overrightarrow{\mathbf{x}_1^*} \ne \overrightarrow{\mathbf{x}_2^*}$, if $\vec{f}(\overrightarrow{\mathbf{x}_1^*}) = \vec{f}(\overrightarrow{\mathbf{x}_2^*})$, then they are equivalent Pareto optimal solutions to each other.

Multimodal multi-objective optimization problem: For a multi-objective optimization problem in equation (1), if there exist equivalent Pareto optimal solutions of interest, then it is regarded as a multimodal multi-objective optimization problem.

Pareto optimal solution set: Subject to satisfying constraints, the set of all Pareto optimal solutions is the Pareto optimal solution set.

Constrained Pareto front: The corresponding projection of Pareto optimal solution set in the objective function space is known as the constrained Pareto front.

Unconstrained Pareto front: If we deliberately disregard the problem's constraints, the mapping of the fake optimal solution set obtained in the objective space is referred to as an unconstrained Pareto front.

Given the start, goal, necessary vertices set, and congestion vertices set, the purpose of the multimodal multi-objective multi-point shortest path planning problem is to find all the Pareto optimal solutions of the optimal shortest routes from the start to the end while passing through all necessary points. The definition and detailed description of the MMMSPP are provided below.

TABLE I
FREQUENTLY USED NOTATIONS

Notation	Description
Gen	Maximum number of generations
N	Population size
Z	Map for path planning
D	Dimensions of decision variables
$Z_{i,j}$	The point located at abscissa i and ordinate j on the city map Z
OP	Optimal paths
V	Set of reduction vertices
E	Set of reduction edges
NE	Set of necessary vertices
P	Parent population
Q	Offspring population
CPF	Pareto front satisfying the constraints
CO	Set of congestion vertices
v	Reduction vertex
e	Reduction edge
π	Path
$\prec$	Dominance
$\overrightarrow{fd}(\cdot)$	Degree of congestion from all directions
$fi(\cdot)$	Number of intersection points
$fl(\cdot)$	Length of the path
$fc(\cdot)$	Number of congestion vertices of the path
$h(\cdot)$	Necessary point constraint

Graph representation: Let $G = (V, E)$ denote a finite undirected graph with $|V|$ vertices and $|E|$ edges. Every edge $e = (v_i, v_j) \in E$ starts at $v_i \in V$ and ends at $v_j \in V$. $NE \subseteq V$ denotes the necessary vertices set, while $CO \subseteq V$ represents the congestion vertices set.

Path representation: Let v_1 represents the start, v_l represents the goal, and $\pi(v_1, v_l) = \{v_1, v_2, \ldots, v_{l-1}, v_l\}$ denotes a path that consists of a list of vertices with each pair of adjacent vertices $v_k, v_{k+1}, k \in \{1, 2, \ldots, l-1\}$ connected by an edge $(v_k, v_{k+1}) \in E$. A path can also be represented by its compound edges $\pi(v_1, v_l) = \{e_1, e_2, ., e_{l-1}\}$, where it holds $\forall k \in \{1, \ldots, l-1\} : e_k = (v_k, v_{k+1}) \in E$.

Objective functions: For a given path $\pi(v_1, v_l)$, the total degree of congestion objective values when moving from v_1 to v_l along π is expressed as: $\overrightarrow{fd}(\pi(v_1, v_l)) = \sum_{k=1}^{k=l} \overrightarrow{fd}(v_k)$, where an objective vector $\overrightarrow{fd}(v_k) \in R^M$, $v_k \in V$ represents the degree of congestion passing through the vertex v_k from all directions. Additionally, the number of intersection points along the path is denoted as $fi(\pi(v_1, v_l))$ and the length of the path is represented by $fl(\pi(v_1, v_l)) = \sum_{k=1}^{k=l-1} fl(e_k)$. Finally, the number of congestion vertices in the path is expressed as $fc(\pi(v_1, v_l)) = \sum_{k=1}^{k=l} es(v_k, CO), k \in \{1, 2, \ldots, l\}$, where $es(v_k, CO)$ is a a binary function indicating whether a vertex v_k belongs to the congestion set CO, *ie*., $es(v_k) = 1$ if $v_k \in CO$ and $es(v_k) = 0$ otherwise.

Constraints: A path $\pi(v_1, v_l)$ is considered feasible if it passes through all necessary vertices in the set NE. The required point is treated as an equality constraint, ensuring that each required point is included in the path. This can be expressed as: $h(\pi(v_1, v_l)) : |NE| - \sum_{k=1}^{k=|NE|} es(ne_k, \pi) = 0$. Here, $es(ne_k, \pi)$ is the existence function ensuring that each required point in NE is visited by the path π.

IV. PROPOSED ALGORITHM

In this section, the framework of the proposed MMOEA-CDP is initially presented in Section IV-A.

Algorithm 1 Framework of the Proposed MMOEA-CDP

Input: *Gen* (Maximum number of generations), N (Population size), Z (Map for path planning)

Output: *OP* (Optimal paths)

1 $[V, E] \leftarrow$ *Graph Preprocessing*(Z)
2 $P \leftarrow$ *Initialization*(V, E, N)
3 **while** *termination criterion not fulfilled* **do**
4 $\quad Q \leftarrow$ *Recombination*(P)
5 $\quad [P, CPF] \leftarrow$ *Path similarity-based multimodal selection*$(P \cup Q)$
6 $[P, CPF] \leftarrow$ *Path similarity-based multimodal selection*(P)
7 $OP \leftarrow CPF$

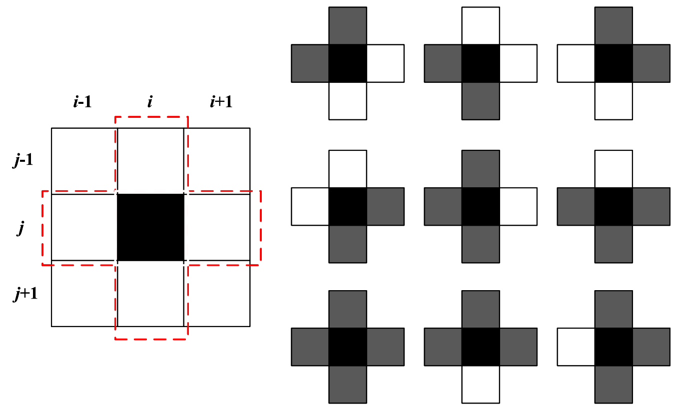


Fig. 4. Illustration of the determination of reduction vertex.

Subsequently, the graph preprocessing is explained in Section IV-B, while the initialization and recombination of the population are described in Section IV-C. Next, we elaborate on the constraint dominance principle-based path comparison strategy and path similarity-based multimodal solutions selection in Section IV-D and Section IV-E respectively, which constitute the key component of MMOEA-CDP. Finally, The time complexity of the proposed algorithm is analyzed in Section IV-F. Table I summarizes the frequently used notations.

A. Framework of the Proposed MMOEA-CDP

The overall framework of MMOEA-CDP is illustrated in Algorithm 1. Firstly, the graph preprocessing method is employed to model the problem's map in line 1, effectively reducing the encoding length of paths while preserving essential map information. Subsequently, population initialization and solution evaluation are conducted in line 2. As long as the current generation is less than the maximum evolutionary generation, crossover and mutation operations are applied to parent population P in order to generate offspring population Q in line 4. Afterwards, a path similarity-based multimodal solutions selection process is performed on the merged population consisting of both parent and offspring populations, resulting in a new population for the subsequent generation in line 5. Finally, upon completion of iterations, the *CPF* of the final population is returned as an optimal set of paths in line 7.

B. Graph Preprocessing

The encoding methodology employed in evolutionary algorithms plays a pivotal role. A well-designed encoding scheme can effectively streamline problem complexity and expedite the solving process. For path planning problems, it is not necessary to consider all areas on the map. In the case of a complex and large-scale map, simplification can be achieved by representing the dot matrix chart as an equivalent reduction graph. This approach effectively reduces the encoding length of individuals in evolutionary algorithms without disregarding important map information. Establishing of an equivalent reduction graph requires constructing distinctive marks, reduction vertices set, and reduction edges set.

1) Constructing Distinctive Marks: The initial step in constructing the reduction graph involves establishing distinct marks. These marks encompass essential information such as the start, goal, necessary points, and areas of congestion within the original city map that are crucial for path planning. Let $Z_{i,j,i\in\{\{1,2,...,Z|\},j\in\{\{1,2...,|Z_i|\}}$ denote the point located at abscissa i and ordinate j on the city map Z. $NE \cup \{Z_{i,j}\}$, if $Z_{i,j}$ is a necessary point, and $CO \cup \{Z_{i,j}\}$, if$Z_{i,j}$ is a congestion point.

2) Building Reduction Vertices: The second step in constructing the reduction graph is to build reduction vertices. Intersections entail waiting for traffic lights and congestion resulting from vehicle lane selection, making the number of intersections a key optimization objective in the MMMSPP problem. While two adjacent intersections can determine an edge, this paper utilizes potential intersections from the original city map as reduction vertices. This allows for the omission of intermediate nodes along this edge in the original map, thereby reducing the decision space and complexity of the problem. Let $Z_{i,j} = True$ if $Z_{i,j}$ is passable else $Z_{i,j} = False$. For the construction of vertices set, $V \cup \{Z_{i,j}\} \Leftrightarrow Z_{i,j} \wedge (Z_{i-1,j} \vee \ Z_{i+1,j}) \wedge (Z_{i,j-1} \vee Z_{i,j+1})$. The formula above requires that $Z_{i,j}$ be passable, and there must be at least one passable point adjacent in both the vertical (up or down) and horizontal (left or right) directions. Fig. 4 illustrates the determination of reduction vertex.

3) Constructing the Reduction Edges: The third step in constructing a reduction graph involves the creation of a set of reduction edges. The pseudo-code for this procedure is shown in Algorithm 2. Firstly, traverse each vertex v in the union of V and NE. Subsequently, it systematically explores the left, right, upward, and downward directions to identify the closest neighboring vertex within the coordinate system established by map Z. This process facilitates the construction of an edge $e(v, Z_{i,j})$. The objective value of the edge is then determined as the summation of the objective values assigned to each point along the edge. Finally, the incorporation of $e(v, Z_{i,j})$ into the set E is executed. The construction of reduction edges is illustrated in Fig. 5, where the gray regions represent passable areas, while the blue point indicate the start location, green point denote the goal location, yellow points signify necessary points, red points represent congestion areas,

Algorithm 2 Construction of the Reduction Edges

```
Input: V (Reduction vertices), NE (Necessary vertices),
       Z (Map for path planning)
Output: E (Reduction edges)
E ← ∅
for v in V ∪ NE do
    for dir in {right, left, up, down} do
        if dir = right then
            i ← v.xaxis + 1, j ← v.yaxis
        else if dir = left then
            i ← v.xaxis − 1, j ← v.yaxis
        else if dir = up then
            i ← v.xaxis, j ← v.yaxis + 1
        else
            i ← v.xaxis, j ← v.yaxis − 1
        cons ← 0, temp⃗ ← 0⃗
        while Z_{i,j} = True do
            temp⃗ ← temp⃗ + fd⃗(Z_{i,j})
            if Z_{i,j} ∈ CO then
                cons ← cons + 1
            if Z_{i,j} ∈ V then
                fd⃗(e(v, Z_{i,j})) ← temp⃗
                if dir = right then
                    fl(e(v, Z_{i,j})) ← i − v.xaxis
                else if dir = left then
                    fl(e(v, Z_{i,j})) ← v.xaxis − i
                else if dir = up then
                    fl(e(v, Z_{i,j})) ← j − v.yaxis
                else
                    fl(e(v, Z_{i,j})) ← v.yaxis − j
                fc(e(v, Z_{i,j})) ← cons
                E ← E ∪ {e(v, Z_{i,j})}
                break
            if dir = right then i ← i + 1
            else if dir = left then i ← i − 1
            else if dir = up then j ← j + 1
            else j ← j − 1
```

black points indicate the reduction vertices, and the black lines with arrows denote the reduction edges corresponding to the vertex v_i.

C. Initialization and Recombination of the Population

1) Initialization of the Population: The initialization of the initial population is demonstrated in Algorithm 3. Firstly, for each new path π, its first vertex is designated as the start location in line 3. Then, until the last vertex $v_{|\pi|}$ in the path π reaches the goal location, the next edge is selected from reduction edges set E as the one where its source is $v_{|\pi|}$ and destination does not appear in path π as shown from line 4 to line 10. Finally, the objective function values of π are ultimately computed in line 11 and subsequently integrated into the population P in line 12.

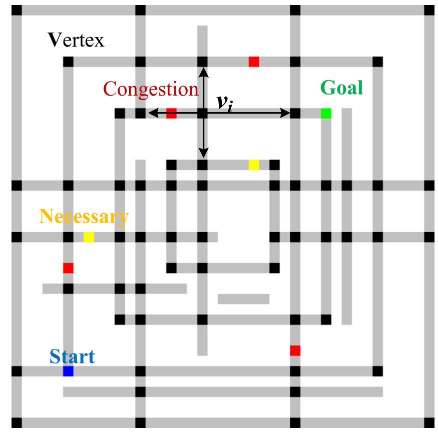


Fig. 5. Illustration of the construction of reduction edges.

Algorithm 3 Initialization of the Population

```
Input: V (Reduction vertices), E (Reduction edges), N
       (Population size)
Output: P (Initial population)
1  P ← ∅
2  while |P| ≤ N do
3      π ← ∅, v_1 ← start, π ← π ∪ {v_1}
4      while v_{|π|} ∈ π ≠ goal do
5          if ∃e ∈ E : e(v_a, v_b) ∧ v_a = v_{|π|} ∧ v_b ∉ π then
6              π ← π ∪ {v_b}, v_{|π'_c|} ← v_b
7          else
8              break
9      if v_{|π|} ≠ goal then
10         continue
11     Calculate the objective functions of π
12     P ← P ∪ π
```

2) Recombination of the Population: In the field of evolutionary computation, recombination operations, such as crossover and mutation, play an indispensable role in generating offspring and enhancing search efficiency. The crossover operator enables exceptional individuals to transmit their superior genes to their progeny, while the mutation operator facilitates the generation of gene fragments absent in the current population, thereby enabling the algorithm to escape local optima. The pseudo-code for the recombination of the population is shown in Algorithm 4. The frequency of crossover and mutation operations is controlled by the probability parameters Pc and Pm, where Pc is set to 0.7 and Pm is set to 0.3. Specifically, the crossover operator employs single-point crossover. Firstly, the junctions are identified in paths π_a and π_b with distinct antecedence vertices in line 5. Then the vertices in π_a and π_b from the junctions to the goal location are exchanged to generate two new individuals

Algorithm 4 Recombination of the Population

Input: P (Parent population)
Output: Q (Offspring population)

1 $Q \leftarrow \emptyset$
2 **while** $|Q| \leq N$ **do**
  /* Crossover */
3   **if** $rand < Pc$ **then**
4     $TempV \leftarrow \emptyset$, $\pi'_a \leftarrow \emptyset$, $\pi'_b \leftarrow \emptyset$
      $\pi_a, \pi_b \leftarrow$ Randomly select two paths from P
5     $TempV \leftarrow \{v_{k \in 1,2,..,|\pi_b|} | v_k \in \pi_a \cap \pi_b, v_{k-1} \in \pi_b \setminus \pi_a\}$
6   **for** v *in* $TempV$ **do**
7       $j \leftarrow index(v, \pi_a)$
8       $\pi'_a \leftarrow \{v_i | i \leq j\} \in \pi_a \cup \{v_i | i > k\} \in \pi_b$
9       $\pi'_b \leftarrow \{v_i | i \leq k\} \in \pi_b \cup \{v_i | i > j\} \in \pi_a$
10   Calculate the objective functions of π'_a and π'_b
11       $Q \leftarrow Q \cup \pi'_a \cup \pi'_b$
  /* Mutation */
12   **if** $rand < Pm$ **then**
13   **for** dir *in {backward, forward}* **do**
14       $\pi_c \leftarrow$ Randomly select one path from P
15       $\pi'_c \leftarrow \emptyset$, $aim \leftarrow \emptyset$
16   **if** $dir = backward$ **then**
17         $v_{r \in [1, |\pi_c| - 2]} \leftarrow$ Randomly select from π_c
18         $\pi'_c \leftarrow \{v_i | i \leq r\} \in \pi_c$
19         $aim \leftarrow goal$
20   **else**
21         $v_{r \in [3, |\pi_c|]} \leftarrow$ Randomly select from π_c
22         $\pi'_c \leftarrow \{v_i | i \geq r\} \in \pi_c$, $\pi'_c \leftarrow reverse(\pi'_c)$
23         $aim \leftarrow start$
24   **while** $v_{|\pi'_c|} \in \pi'_c \neq aim$ **do**
25   **if** $\exists e \in E : e(v_a, v_b) \wedge v_a = v_{|\pi'_c|} \wedge v_b \notin \pi'_c$ **then**
26           $\pi'_c \leftarrow \pi'_c \cup \{v_b\}$, $v_{|\pi'_c|} \leftarrow v_b$
27   **else**
28   **break**
29   **if** $v_{|\pi'_c|} \neq aim$ **then**
30   **continue**
31   **if** $dir = forward$ **then**
32         $\pi'_c \leftarrow reverse(\pi'_c)$
33   Calculate the objective functions of π'_c
34       $Q \leftarrow Q \cup \pi'_c$

π'_a and π'_b in line 8 and 9. The mutation operator adopts a multi-point segmentation mutation, including backward and forward modes. Firstly, for a given path π_c, randomly select a non-start or non-goal vertex v_r as the mutation vertex as shown in line 17 and 21. Then, the backward mutation will reconstruct the path from vertex v_r to the goal location, while the forward mutation will reconstruct the path from vertex v_r to the start location.

D. Constraint Dominance Principle-Based Path Comparison Strategy

In MMMSPP, necessary points are typically treated as critical constraints that must be satisfied. Current algorithms commonly adopt two main approaches to handling the necessary point constraint: strict constraint satisfaction and segmented optimization. Strict constraint satisfaction considers any candidate path that fails to meet the necessary point constraint as an infeasible solution, which is then directly eliminated from the solution set. While straightforward, this method is overly rigid and may overlook potentially optimal solutions that partially satisfy the necessary point constraint. As a result, this strategy restricts the algorithm's ability to explore infeasible regions and converge towards the true CPF. Segmented optimization treats necessary points as breakpoints, dividing the problem into multiple sub-paths. Each sub-path is calculated independently as a shortest path, and the final solution is obtained by concatenating these sub-paths. However, this method has notable limitations in multimodal multi-objective optimization. First, it often leads to a loss of global optimality, as the independent computation of sub-paths focuses on local optima, neglecting the overall optimality of the full path. Second, it struggles to maintain solution diversity, typically producing a single optimal path and failing to capture multiple equivalent solutions in multimodal scenarios.

To address the limitations of conventional methods in handling the necessary point constraint, propose the CDP-based path comparison strategy is proposed. This strategy introduces a relaxation mechanism into the traditional principle of dominance that permits the inclusion of infeasible solutions by evaluating their quality based on the number of necessary points traversed. Unlike conventional approaches that rigidly eliminate infeasible paths, CDP leverages this relaxation to maintain a balance between solution feasibility and quality, thereby enhancing the algorithm's ability to explore the decision space and converge towards the CPF. The strategy operates in several stages. First, it assesses the feasibility of candidate paths by determining whether they traverse all necessary points. Paths that meet all constraints are deemed feasible, while those that do not are classified as infeasible. When comparing a feasible path with an infeasible one, priority is given to the feasible path. In cases where both paths are feasible, their dominance relationship is determined using traditional multi-objective dominance rules. Conversely, if both paths are infeasible, CDP evaluates their quality based on the number of necessary points traversed, granting dominance to the path that passes through more necessary points. This systematic mechanism is formalized in Algorithm 5, where each step is carefully designed to address various scenarios encountered in path comparison.

The advantages of CDP are multifaceted. By relaxing the necessary point constraint, the strategy enables the exploration of infeasible regions, expanding the algorithm's search capability in the decision space. This flexibility allows the algorithm to navigate towards high-quality solutions that may otherwise be disregarded. Furthermore, CDP improves the diversity of solutions by utilizing infeasible paths to guide the search process, ensuring the algorithm maintains a diverse

Algorithm 5 Constraint Dominance Principle-Based Path Comparison Strategy

Input: π_a (Candidate path), π_b (Candidate path), NE (Necessary vertices set)

Output: $CDom$ (Boolean value whether π_a constrained dominates π_b)

$nca \leftarrow 0$, $ncb \leftarrow 0$
$nca \leftarrow |\pi_a \cap NE|$, $ncb \leftarrow |\pi_b \cap NE|$
if $h(\pi_a) = 0 \wedge h(\pi_b) \neq 0$ **then**
  $CDom \leftarrow$ ***True***
else if $h(\pi_a) = 0 \wedge h(\pi_b) = 0$ **then**
  if $\pi_a \prec \pi_b$ **then**
    $CDom \leftarrow$ ***True***
  else
    $CDom \leftarrow$ ***False***
else if $h(\pi_a) \neq 0 \wedge h(\pi_b) \neq 0$ **then**
  if $nca > ncb$ **then**
    $CDom \leftarrow$ ***True***
  else
    $CDom \leftarrow$ ***False***
else
  $CDom \leftarrow$ ***False***

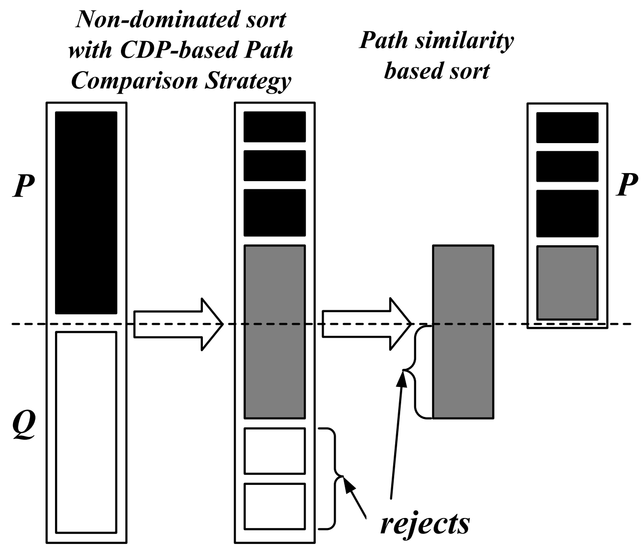


Fig. 6. Illustration of the path similarity-based multimodal solutions selection strategy.

set of candidate paths—a critical requirement for multimodal optimization problems.

E. Path Similarity-Based Multimodal Solutions Selection

Current MOEAs often exhibit limited effectiveness in solving MMMSPP due to their predominant focus on maintaining diversity in the objective space while neglecting the distribution of solutions in the decision space. This oversight frequently results in the loss of equivalent Pareto optimal solutions, as structurally diverse solutions with identical objective values are often discarded. Additionally, the lack of explicit mechanisms to maintain decision-space diversity leads to incomplete exploration, with certain regions being over-explored while others remain underexplored, exacerbating premature convergence. Moreover, most existing multimodal optimization methods are designed for continuous real-valued problems, making them less suitable for combinatorial scenarios like MMMSPP, where path structures are critical for capturing decision-space diversity and achieving comprehensive exploration.

To address the challenges of MMMSPP, the path similarity-based multimodal solutions selection strategy is proposed, which integrates a path similarity metric into the conventional non-dominated sorting framework. Unlike traditional methods that rely on objective-space diversity indicators, this strategy prioritizes the diversity of solutions in the decision space. The workflow of the proposed strategy is depicted in Fig.6. The strategy begins by employing non-dominated sorting with CDP-based path comparison strategy to organize the population of candidate paths into multiple ranks based on their dominance relationships in the presence of constraints. The constrained non-dominated front obtained from this sorting process is stored in the CPF, representing high-quality solutions that achieve both feasibility and convergence. When the population size exceeds the predefined maximum, the selection process focuses on maintaining decision-space diversity among the solutions in the last rank. At this stage, the path similarity metric replaces the traditional crowding distance as the diversity indicator. The formula for calculating path similarity is presented as follows:

$$PathSimilarity\,(\pi_i) = \frac{\sum_{j=1}^{|rank|} card\left(\pi_i \cap \pi_j\right)}{card\,(\pi_i)} \qquad (3)$$

where $|rank|$ denotes the number of paths in the same rank, the $card$ represents a counting function, and $card(\pi_i \cap \pi_j)$ signifies the number of common vertices between two paths. Solutions with lower $PathSimilarity$ values, indicating higher diversity in the decision space, are retained

This strategy adheres to the basic workflow of non-dominated sorting. The refined solutions from the last rank, selected based on their path similarity, are combined with higher-ranked solutions to form the next generation's population. This process systematically balances feasibility, convergence, and diversity, ensuring that the algorithm effectively captures the multimodal nature of MMMSPP. The pseudo-code of Algorithm 6 illustrates these steps, where lines 2-14 describe the application of non-dominated sorting and CPF construction and lines 16-34 focus on refining the last rank by evaluating and retaining solutions based on their path similarity metric.

Through this comprehensive approach, the strategy directly addresses the limitations of traditional methods. By focusing on decision-space diversity, it ensures that structurally distinct solutions are preserved, even when their objective values are identical. This not only enhances the algorithm's ability to explore underrepresented regions of the decision space but also ensures a more complete and representative PF for MMMSPP.

Algorithm 6 Path Similarity-Based Multimodal Solutions Selection

Input: P (Population of candidate paths)
Output: P' (New population of candidate paths), *CPF* (Constrained Pareto front)

$\overrightarrow{Do} \leftarrow \emptyset$, $\overrightarrow{ND} \leftarrow 0$, $P' \leftarrow \emptyset$, $front \leftarrow \emptyset$, $CPF \leftarrow \emptyset$, $temp \leftarrow \emptyset$
for $\pi_i \in P$ *with* $i \in \{1, 2, \ldots, |P|\}$ **do**
  for $\pi_j \in P$ *with* $j \in \{i+1, \ldots, |P|\}$ **do**
    if *CDP-based path comparison strategy*(π_i, π_j) **then**
      $Do_i \leftarrow Do_i \cup \{\pi_j\}$, $ND_j \leftarrow ND_j + 1$
    if *CDP-based path comparison strategy*(π_j, π_i) **then**
      $Do_j \leftarrow Do_j \cup \{\pi_i\}$, $ND_i \leftarrow ND_i + 1$
  if $ND_i = 0$ *and* $|P'| < N$ **then**
    $P' \leftarrow P' \cup \{\pi_i\}$, $front \leftarrow front \cup \{\pi_i\}$
$CPF \leftarrow front$
while $|front| \neq 0$ **do**
  for $\pi \in front$ **do**
    $i \leftarrow index(\pi, P)$
    for *each* π' *in* Do_i **do**
      $j \leftarrow index(\pi', P)$, $ND_j \leftarrow ND_j - 1$
      if $ND_j = 0$ **then**
        $temp \leftarrow temp \cup \{\pi'\}$
  $front \leftarrow temp$, $temp \leftarrow \emptyset$
  if $|P'| + |front| \leq N$ **then**
    $P' \leftarrow P' \cup front$
  else
    for $\pi'' \in front$ **do**
      Calculate path similarity of π''
    Sort $front$ by path similarity in ascending order
    $P' \leftarrow P' \cup \{\pi''_i$ for $i = 1, 2, \ldots, |N - |P'||\}$

F. Complexity Analysis

In this section, the computational complexity of one generation of the proposed MMOEA-CDP is analyzed. During the *Graph Preprocessing* phase, the proposed method for constructing reduction vertices and reduction edges significantly reduces the dimensionality of decision variables in the solution encoding process. The time complexity of the *Initialization* phase is determined by the population size and the dimensionality of the decision variables, resulting in $O(ND)$. Similarly, the *Recombination phase*, which includes crossover and mutation operations applied to each individual in the population, also has a time complexity of $O(ND)$. The *CDP-based path comparison strategy* involves two primary sources of computational complexity. First, it requires determining whether each node in a path belongs to the set of necessary vertices NE. When using hash tables, this operation has a time complexity of $O(|NE|)$. Second, it involves evaluating dominance relationships between individuals, which has a time complexity of $O(M)$, where M represents the number of objectives. Consequently, the overall time complexity of the *CDP-based path comparison strategy* is $O(|NE| + M)$. The time complexity of the *Path similarity-based multimodal solutions selection* primarily stems from two components, one is the non-dominated sorting process integrated with the *CDP-based path comparison strategy*, which has a time complexity of $O(N^2(|NE| + M))$, and the other is the calculation of path similarity, with a time complexity of $O(N^2 D)$. Excluding the *Initialization* phase, the computational complexity of one generation of the proposed MMOEA-CDP is $O(ND) + O(N^2(|NE| + M + D))$. According to Big-O notation, this can be simplified to $O(N^2(|NE| + M + D))$.

V. Experimental Setting

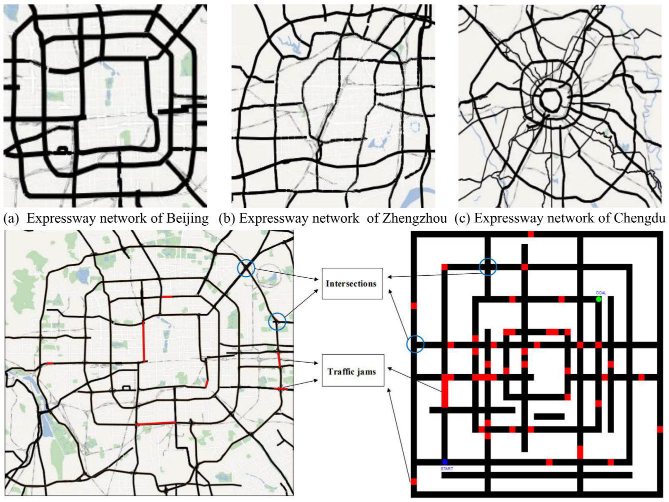
The experimental setup has been meticulously designed to address three distinct research questions (RQs) and provide empirical evidence, aiming to illuminate the performance of MOEAs in tackling the MMMSPP problem under investigation.

- RQ1: Which is the best approach for solving MMMSPP?
- RQ2: Do the proposed constraint dominance principle-based path comparison strategy and the path similarity-based multimodal solutions selection strategy enhance the overall algorithmic efficiency?
- RQ3: What level of impact can be expected from incorporating the required points as constraints in addressing the multimodal multi-objective path planning problem?

To address RQ1, the performance of the proposed MMOEA-CDP is compared with state-of-the-art algorithms on benchmark test suites, their time complexities are analyzed, and its practical effectiveness is validated through a case study on real-world instances in Section VI-A. In order to tackle RQ2, an ablation experiment will be conducted to assess the performance of the proposed algorithm in Section VI-B. To investigate RQ3, we will control the number of necessary points to elucidate the impact of this constraint on problem complexity in Section VI-C.

A. Test Problems

The algorithm's ability to solve MMMSPP problems is comprehensively evaluated from three perspectives: multi-objective, multimodal, and constrained. To achieve this, three categories of test problems are meticulously designed. The type-I problems are relatively straightforward and evaluate the algorithm's capacity to preserve equivalent global optimal solutions for a limited number of objective functions. The type-II problems examine the algorithm's capability to tackle many-objective (the number of objective functions more than three) [46] path planning problems, wherein the number of objective functions is progressively augmented. The type-III problems involve necessary points constraint. The problem suite is derived from the 2021 IEEE CEC path planning competition [45], and the maps are adapted from real transportation networks in Chinese cities including Beijing, Zhengzhou, and Chengdu. Fig. 7 illustrates the maps used in the test suite. Information and features of the MMMSPP test suite are given in detail in Table II.



(a) Expressway network of Beijing (b) Expressway network of Zhengzhou (c) Expressway network of Chengdu

(d) The expressway network of Beijing and the map case in the test suite

Fig. 7. Illustration of the map in test suite.

TABLE II
DESCRIPTION AND FEATURES OF THE MMMSPP TEST SUITE

Type	Feature	Problem	Objectives	Number of Objectives	Number of necessary points
I	Multimodal, Multi-objective	1	$fl(\cdot), fc(\cdot)$	2	–
		2	$fl(\cdot), fc(\cdot), fi(\cdot)$	3	–
		3	$fl(\cdot), fc(\cdot), fi(\cdot)$	3	–
		4	$fl(\cdot), fc(\cdot), fi(\cdot)$	3	–
		5	$fl(\cdot), fc(\cdot), fi(\cdot)$	3	–
II	Multimodal, Many-objective	6	$fl(\cdot), fd_1(\cdot)$	2	–
		7	$fl(\cdot), \overrightarrow{fd_{\{1,2\}}}(\cdot)$	3	–
		8	$fl(\cdot), \overrightarrow{fd_{\{1,2,3\}}}(\cdot)$	4	–
		9	$fl(\cdot), \overrightarrow{fd_{\{1,2,\ldots,4\}}}(\cdot)$	5	–
		10	$fl(\cdot), \overrightarrow{fd_{\{1,2,\ldots,6\}}}(\cdot)$	7	–
III	Multimodal, Multi-objective, Constrained	11	$fl(\cdot), fd_1(\cdot)$	2	1
		12	$fl(\cdot), \overrightarrow{fd_{\{1,2\}}}(\cdot)$	3	2

B. Compared Algorithms

The performance of the proposed MMOEA-CDP is validated through a comparative analysis with five state-of-the-art algorithms: ClusteringGA [57], DNEA [38], MDACO [44], MMEA-SES [42], and MACOSX [43]. These algorithms are specifically designed to address MMMSPP and were participants in the IEEE CEC 2021 path planning competition [45]. ClusteringGA [57] employs a clustering-based evolutionary framework that divides the population into subpopulations, effectively enhancing decision-space diversity and enabling the identification of multiple equivalent constrained Pareto sets. DNEA [38] leverages a dual-niching strategy that integrates niche-sharing mechanisms in both decision and objective spaces, ensuring diverse solution sets by mitigating overlap in the decision space. MDACO [44] adopts a multi-objective ant colony optimization approach, initializing pheromones with Dijkstra's algorithm and employing an adaptive pheromone adjustment strategy to preserve decision-space diversity. MMEA-SES [42] introduces a customized crowding distance calculation and diversity-based fitness evaluation tailored for MMMSPP, enhancing decision-space diversity while maintaining convergence quality. MACOSX [43] integrates a topological map-based preprocessing strategy into the NSGA-II [30] framework, simplifying grid maps to reduce computational complexity and improve optimization efficiency.

C. Parameter Settings

The parameter settings of the comparison algorithms remain consistent with those reported in the original papers. In most problems, the maximum number of generations Gen parameter is typically set to 100, while in problems 5, 10, and 12, it is adjusted to values of 1000, 500, and 200 respectively. Similarly, the population size N parameter is usually set to a value of 100 across most problems; however, for problems 5, 9, 10 and 12 it is modified to values of 1500, 200, 2000, and 2000 respectively. To ensure fairness, the final experimental results for all comparison algorithms represent the average of 30 independent runs.

D. Performance Indicator

To assess the performance of MOEAs, numerous metrics have been proposed, such as the inverted generational distance (IGD) [47], hypervolume (HV) [48], and additive epsilon (EPS_+) [49] indicators. However, in the context of MMMSPP problems, the purpose is to identify all Pareto optimal paths that may exhibit equivalent objective values. Consequently, these metrics are unsuitable for evaluating the performance of MOEAs on the MMMSPP test suite. Subsequently, the metric of the number of distinct optimal solutions (NOS) [42] is employed as the indicator to evaluate the performance. The mathematical representation of NOS is depicted as follows:

$$NOS = card(OPS \cap TPS) \tag{4}$$

where OPS represents the Pareto solution set obtained by the algorithm, TPS represents the true Pareto optimal solution set of the problem, and NOS denotes the number of common solutions in OPS and TPS. A higher value of NOS indicates a superior quality of the solution set derived from the algorithm. Additionally, the Wilcoxon rank sum tests are employed with a significance level of 0.05 to assess the statistical significance of differences. The symbols "+", "–", and "≈" denote that the compared algorithm performs significantly better than, worse than, or equivalently to the proposed MMOEA-CDP. The source codes of the proposed MMOEA-CDP for MMMSPP are downloadable from https://github.com/JaywayXu/MMOEA-CDP.

VI. RESULTS AND ANALYSIS

A. RQ1: Effectiveness of the MMOEA-CDP Approach

1) Performance on Benchmark Test Suite: To validate the efficacy of the proposed approach, the proposed

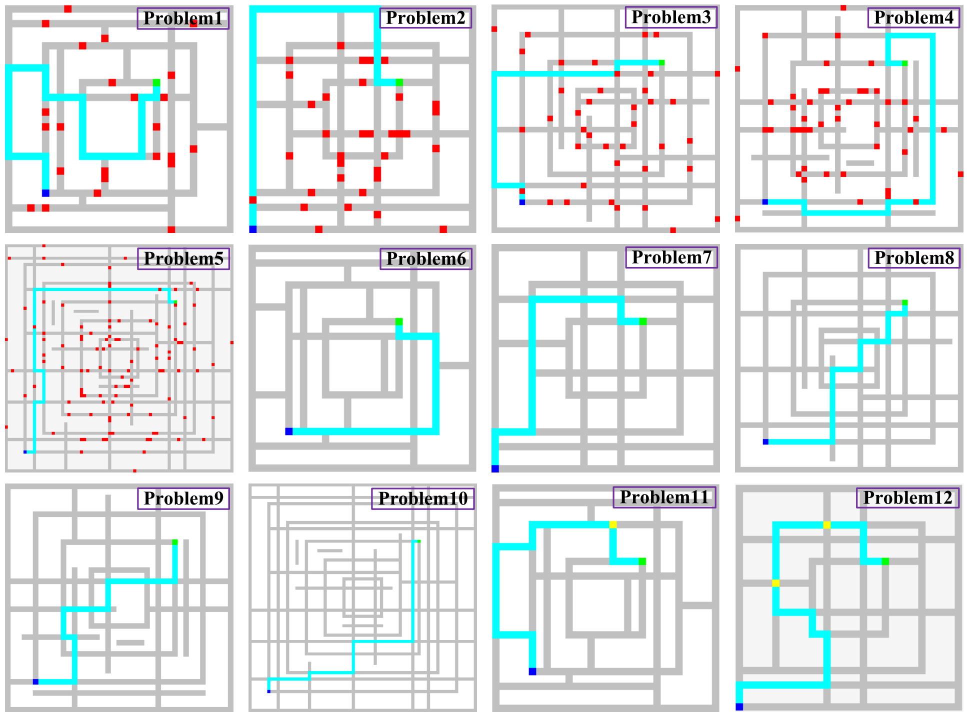


Fig. 8. Illustration of one of the optimal solutions for each problem. In type-I problems, the congested vertices are represented by red points. In type-II problems, the degree of congestion is indicated by congestion functions. In type-III problems, the necessary points are denoted by yellow points.

MMOEA-CDP is compared with five state-of-the-art algorithms. Table III presents the average and standard deviation NOS values and obtained by MMOEA-CDP and the compared algorithms on MMMSPP test suite, where thirty independent runs are conducted for each algorithm on each test instance. It is evident from the statistical results of Wilcoxon rank sum test in the last row of Table III that MMOEA-CDP demonstrates superior performance across all test instances. And the proposed MMOEA-CDP is capable of identifying all the optimal solutions. Conversely, the remaining algorithms cannot perform well on all the test instances.

To be specific, ClusteringGA exhibits poor performance across all test instances, primarily because its clustering-based approach is not well-suited for the structural characteristics of paths. The recombination process is confined to niches formed within individual clusters, which significantly restricts the algorithm's exploratory capability in the decision space and makes it prone to premature convergence to local optima. DNEA performs worse than the proposed MMOEA-CDP on 9 out of 12 test instances. This is attributed to its neglect of the unique structural characteristics of path encoding. By relying solely on a simplistic linear combination of decision-space niche ranges and objective-space crowding distances as a diversity indicator, DNEA fails to effectively preserve decision-space diversity. MDACO performs worse than the proposed MMOEA-CDP on 5 out of 12 test instances. Although it employs an ant colony optimization algorithm as its search engine, its adaptive pheromone adjustment method proves ineffective in handling multimodal multi-objective problems with a large number of equivalent optimal solutions, such as instances 5, 8, 9, 10, and 12. MMEA-SES improves the environment selection strategy, thereby exhibiting superior performance on most multimodal problems. However, as the number of objectives increases, it fails to address the selection pressure arising from many-objective scenarios, resulting in suboptimal performance on Problems 9 and 10 with more than four objectives. MACOSX is able to find all equivalent optimal solutions in only 2 out of the 12 test instances. This limitation arises because it relies on crowding distance in the objective space for selecting superior individuals while neglecting diversity in the decision space, which is crucial for capturing all equivalent solutions in multimodal multi-objective problems.

The noteworthy aspect is that the proposed MMOEA-CDP exhibits superior performance compared to other algorithms on test instances 5, 9, and 10, with over 20 equivalent optimal solutions. The reason is that the proposed path similarity-based multi-modal solution selection mechanism selects the optimal solutions based on the distance between individuals in the decision space, allowing it to retain all equivalent Pareto optimal solutions. MMOEA-CDP also demonstrates exceptional performance in instances involving constraints on necessary points, such as problem 11 and 12. The reason is that the proposed path comparison strategy based on the constraint dominance principle allows for relaxation of constraints and acceptance of partially satisfying infeasible solutions, depending on the level of constraint satisfaction. Consequently, this approach empowers the population to span large infeasible regions. Fig. 8 illustrates one optimal solution obtained by the proposed MMOEA-CDP on each test instance. The Fig. 9 illustrates the complete set of optimal solutions for Problem 1 obtained through MMOEA-CDP. From the results, it can be observed that MMOEA-CDP effectively preserves all optimal solutions even when they possess identical objective values.

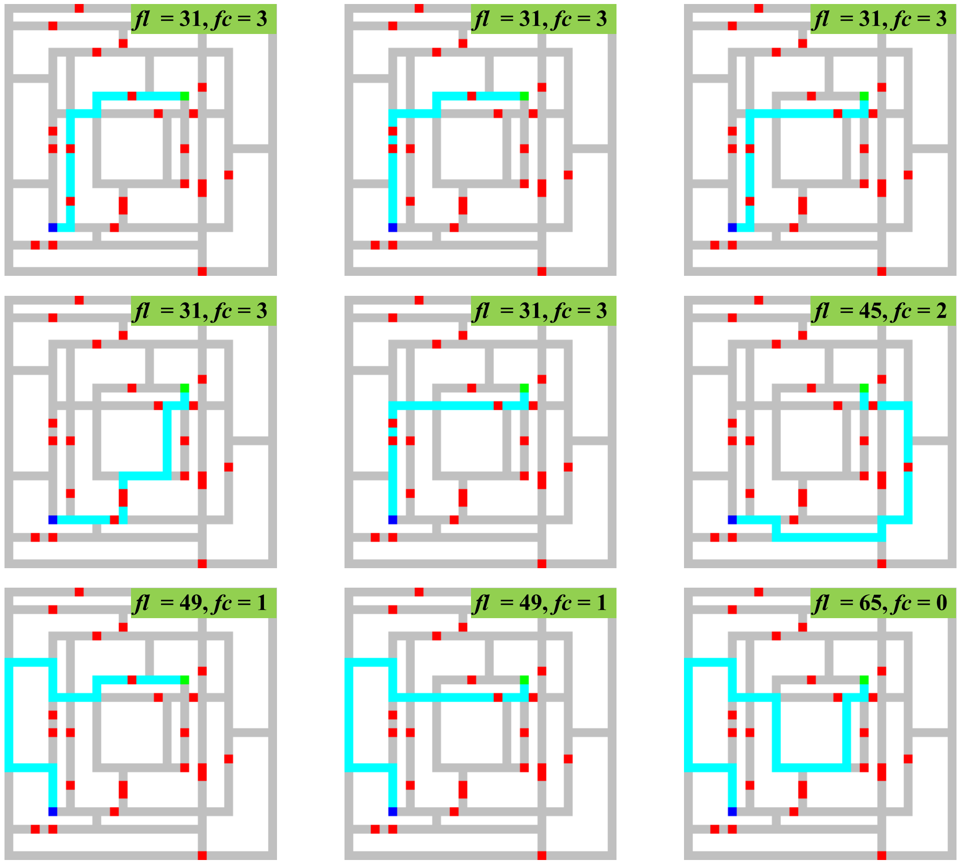


Fig. 9. All different optimal solutions for problem 1, where the objective vectors are (31,3), (45,2), (49,1), and (65,0) for the 1-5th, 6th, 7-8th, and 9th subfigures, respectively.

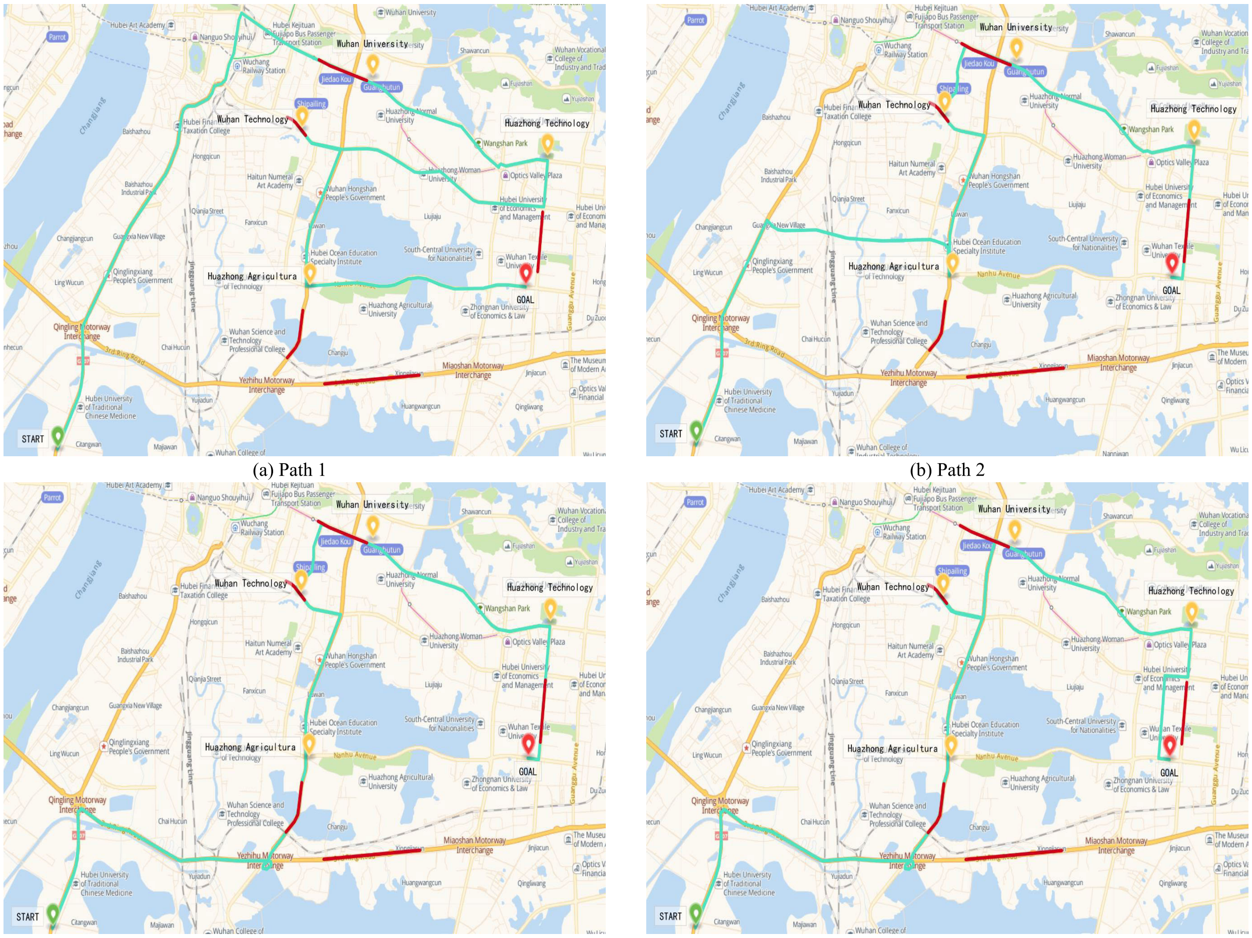
TABLE III
THE AVERAGE AND STANDARD DEVIATION OF NOS VALUES OBTAINED BY MMOEA-CDP AND THE COMPARED ALGORITHMS ON MMMSPP TEST SUITE OVER 30 INDEPENDENT RUNS AND THE NUMBER OF OPTIMAL SOLUTIONS IN THE TRUE PS. BEST RESULT IS MARKED IN GRAY

Problem	ClusteringGA	DNEA	MDACO	MMEA-SES	MACOSX	MMOEA-CDP	True PS
1	6.43- (0.93)	9.00≈ (0.00)	9.00 ≈ (0.00)	9.00 ≈ (0.00)	9.00 ≈ (0.00)	9.00 (0.00)	9
2	14.67- (2.96)	24.00≈ (0.00)	24.00 ≈ (0.00)	24.00 ≈ (0.00)	22.32- (2.12)	24.00 (0.00)	24
3	12.56- (0.83)	10.03- (2.30)	13.00 ≈ (0.00)	13.00 ≈ (0.00)	12.54- (0.85)	13.00 (0.00)	13
4	6.53- (1.36)	9.00 ≈ (0.00)	9.00 ≈ (0.00)	9.00 ≈ (0.00)	8.85- (0.34)	9.00 (0.00)	9
5	4.32- (3.65)	17.33- (1.83)	18.47- (3.53)	18.67- (3.63)	16.73- (2.21)	24.00 (0.00)	24
6	4.26- (0.92)	4.80- (0.40)	5.00 ≈ (0.00)	5.00 ≈ (0.00)	4.76- (0.56)	5.00 (0.00)	5
7	6.22- (2.36)	15.67- (0.47)	16.00 ≈ (0.00)	16.00 ≈ (0.00)	14.32- (1.04)	16.00 (0.00)	16
8	31.20- (4.92)	44.59- (3.51)	43.25- (3.62)	48.00 ≈ (0.00)	41.86- (4.73)	48.00 (0.00)	48
9	65.16- (9.42)	86.43- (4.43)	102.58- (2.35)	104.76- (1.35)	103.48- (1.78)	105.00 (0.00)	105
10	427.83- (31.15)	582.43- (32.82)	1045.73- (44.21)	1276.57- (4.42)	992.65- (49.12)	1280.00 (0.00)	1280
11	3.44- (0.42)	3.63- (0.43)	4.00 ≈ (0.00)	4.00 ≈ (0.00)	4.00 ≈ (0.00)	4.00 (0.00)	4
12	7.38- (1.44)	6.67- (2.43)	18.67- (1.81)	22.00 ≈ (0.00)	20.23- (1.83)	22.00 (0.00)	22
+/-/≈	0/12/0	0/9/3	0/5/7	0/3/9	0/10/2		

While traditional MOEAs excel in identifying the Pareto front, discovering all equivalent solutions remains a formidable challenge.

2) Comparison of Time Complexity: To illustrate the computational efficiency of the proposed MMOEA-CDP, Table IV analyzes and compares the time complexities of the proposed algorithm with several state-of-the-art approaches. The time complexity of these algorithms primarily stems from three components: convergence enhancement strategies, diversity maintenance strategies, and feasibility satisfaction strategies. Regarding convergence enhancement, both the MMOEA-CDP and the comparison algorithms employ non-dominated sorting, resulting in a time complexity of $O(N^2M)$, where all algorithms perform equivalently.

For diversity maintenance, ClusteringGA adopts a clustering-based approach with a time complexity of $O(N^3)$, which becomes inefficient as the population size increases. DNEA, MDACO, MMEA-SES, and MACOSX consider



(a) Path 1 (b) Path 2

(c) Path 3 (d) Path 4

Fig. 10. Optimal paths identified by MMOEA-CDP for the real-world road network of Wuhan during morning peak hours.

TABLE IV
TIME COMPLEXITY OF THE PROPOSED MMOEA-CDP ALGORITHM AND THE COMPARED ALGORITHMS

Algorithm	Time Complexity
ClusteringGA	$O(N^3 + N^2M + N\|NE\|D)$
DNEA	$O(N^2(D + 2M) + N\|NE\|D)$
MDACO	$O(N^2(D + 2M) + \|NE\|^2D^2 + \|NE\|!D)$
MMEA-SES	$O(N^2(D + 2M) + \|NE\|^2D^2 + \|NE\|!D)$
MACOSX	$O(N^2(D + 2M) + \|NE\|^2D^2 + \|NE\|!D)$
MMOEA-CDP	$O(N^2(D + M + \|NE\|))$

both decision space and objective space diversity, incurring a time complexity of $O(N^2M + N^2D)$. In contrast, the proposed MMOEA-CDP focuses solely on decision space diversity, avoiding the complexity associated with objective space evaluations, thereby reducing the time complexity to $O(N^2D)$. This design demonstrates a significant efficiency advantage, particularly when the dimensionality of decision variables is high.

In terms of feasibility satisfaction, different strategies are employed to handle necessary point constraints. ClusteringGA and DNEA directly eliminate infeasible solutions by checking whether each individual in the population satisfies the necessary points, with a time complexity of $O(N|NE|D)$. MDACO, MMEA-SES, and MACOSX adopt a segmentation-based approach, dividing paths by necessary points and merging them to construct complete solutions. This method incurs a time complexity of $O(|NE|^2D^2 + |NE|!D)$, where the factorial term $|NE|!$ rapidly increases the computational burden as the number of necessary points grows, significantly limiting scalability. The proposed MMOEA-CDP, however, integrates the CDP strategy to directly evaluate constraint feasibility, achieving a time complexity of $O(N^2|NE|)$ and avoiding the exponential cost associated with factorial operations.

By combining these components, the overall time complexity of MMOEA-CDP is $O(N^2(D + M + |NE|))$. Compared to other algorithms, MMOEA-CDP significantly reduces the computational cost of both diversity maintenance and feasibility satisfaction. This efficiency, combined with its scalability for problems with large numbers of necessary points or high-dimensional decision variables, demonstrates the superior applicability and competitiveness of MMOEA-CDP for solving complex MMMSPP problems.

3) Real-World Case Study: To validate the effectiveness of the proposed MMOEA-CDP in real-world MMMSPP problems, it is applied to the urban road network of Wuhan, China, during weekday morning peak hours. The start point is set at Wuhan University of Science and Technology, and the end

TABLE V

THE AVERAGE AND STANDARD DEVIATION OF NOS VALUES OBTAINED BY THE MOEA, MMOEA, MOEA-CDP, AND ORIGINAL MMOEA-CDP ON THE SELECTED TEST INSTANCES IN MMMSPP TEST SUITE OVER 30 INDEPENDENT RUNS. BEST RESULT IS MARKED IN GRAY

Problem	MOEA	MMOEA	MOEA-CDP	MMOEA-CDP
1	9.00 ≈ (0.00)	9.00 ≈ (0.00)	9.00 ≈ (0.00)	9.00 (0.00)
2	24.00 ≈ (0.00)	24.00 ≈ (0.00)	24.00 ≈ (0.00)	24.00 (0.00)
3	13.00 ≈ (0.00)	13.00 ≈ (0.00)	13.00 ≈ (0.00)	13.00 (0.00)
4	9.00 ≈ (0.00)	9.00 ≈ (0.00)	9.00 ≈ (0.00)	9.00 (0.00)
5	19.00- (3.24)	24.00 ≈ (0.00)	19.00- (3.20)	24.00 (0.00)
6	5.00 ≈ (0.00)	5.00 ≈ (0.00)	5.00 ≈ (0.00)	5.00 (0.00)
7	16.00 ≈ (0.00)	16.00 ≈ (0.00)	16.00 ≈ (0.00)	16.00 (0.00)
8	47.79- (0.99)	48.00 ≈ (0.00)	47.83- (0.90)	48.00 (0.00)
9	104.53- (1.15)	105.00 ≈ (0.00)	104.67- (1.26)	105.00 (0.00)
10	1278.80- (0.54)	1280.00 ≈ (0.00)	1278.67- (0.58)	1280.00 (0.00)
11	3.33- (0.49)	3.42- (0.75)	4.00 ≈ (0.00)	4.00 (0.00)
12	2.17- (1.36)	2.33- (1.33)	21.17- (4.10)	22.00 (0.00)
+/-/≈	0/6/6	0/2/10	0/5/7	

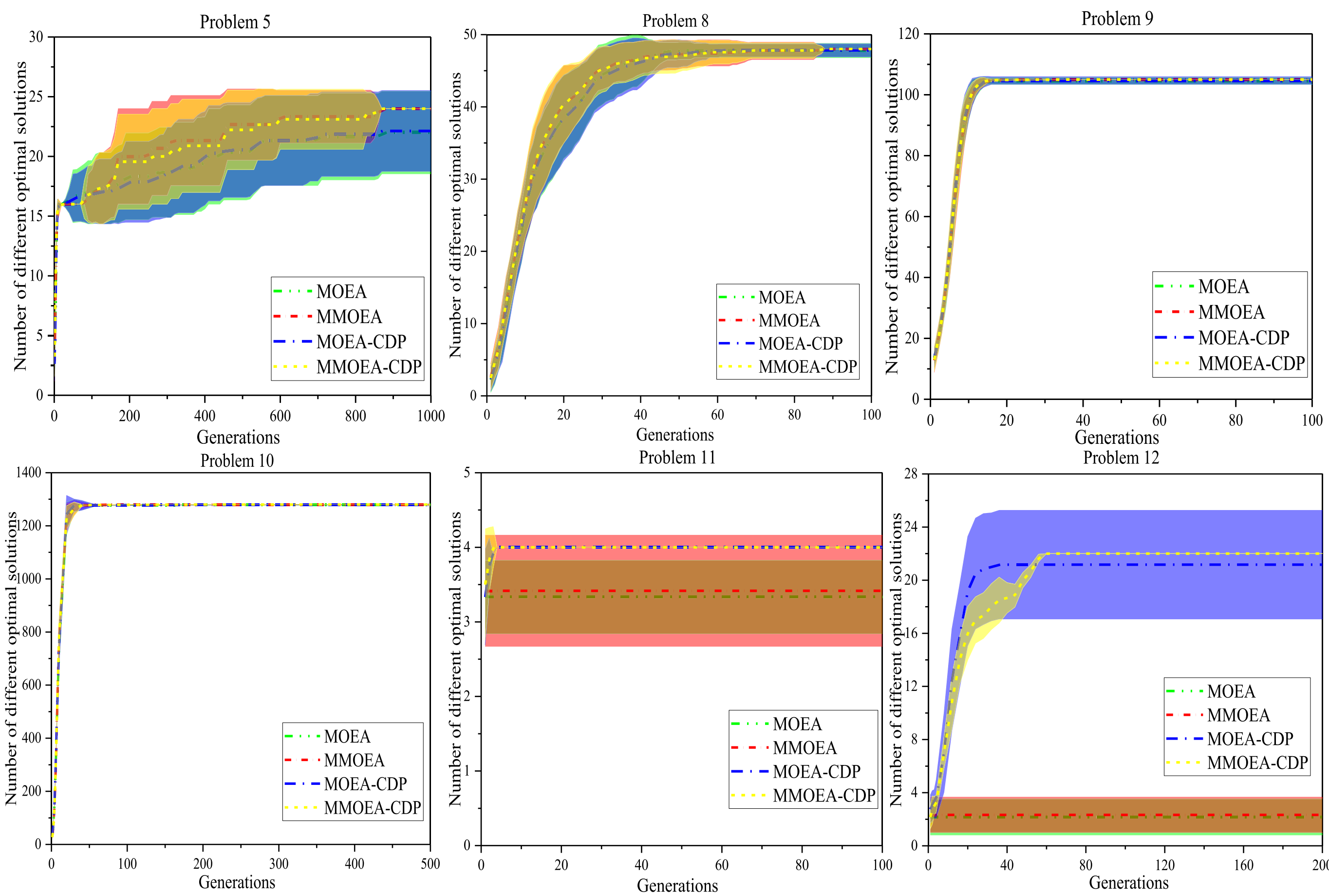


Fig. 11. Convergence profiles and standard deviation of NOS results obtained by the MOEA, MMOEA, MOEA-CDP and original MMOEA-CDP.

point at Wuhan Optics Valley Software Park, with necessary points set at Huazhong Agricultural University, Wuhan University of Technology, Wuhan University, and Huazhong University of Science and Technology. The optimization objectives included minimizing path length, total congestion length, maximizing road speed, minimizing the number of intersections, and minimizing the number of U-turns. To ensure the identification of equivalent optimal solutions and maintain multimodality in the decision space, epsilon-equivalence is employed to introduce an appropriate tolerance for the equality of objective function values. The optimal paths obtained using the proposed MMOEA-CDP are shown in Fig. 10. Among these, Path 1 has the longest length but experiences the shortest congestion length. Paths 2, 3, and 4 have similar lengths; however, Path 3 passes through four congested segments, resulting in the highest congestion length, although it avoids U-turns. Paths 2 and 4 are identified as multimodal optimal solutions due to their similar objective values across all criteria.

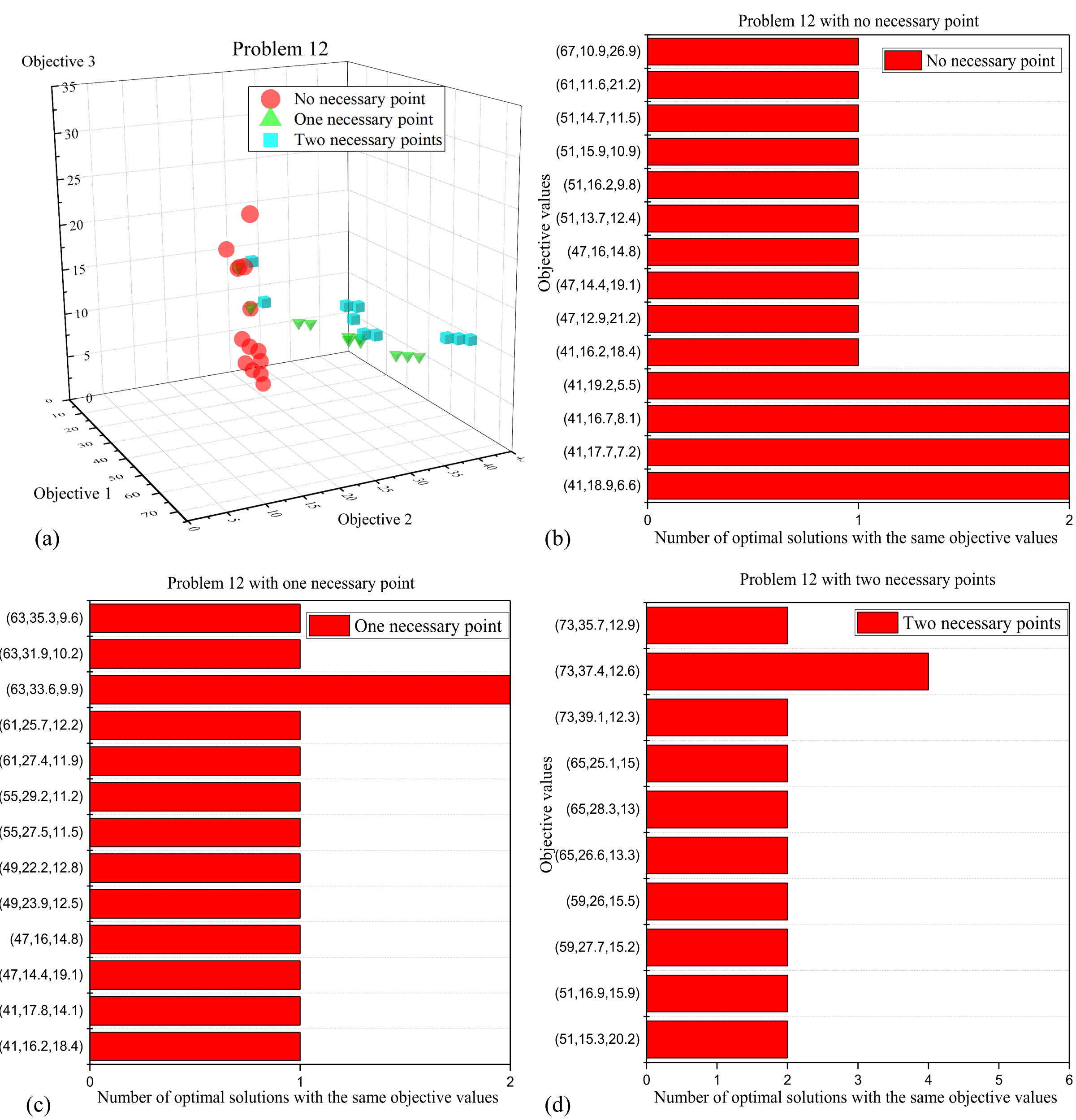


Fig. 12. (a) The Pareto front of the problem 12 in the MMMSPP test suite. (b)-(d) The number of equivalent Pareto optimal solutions corresponding to each point on the Pareto front.

B. RQ2: Ablation Study of the MMOEA-CDP Approach

To verify the effectiveness of the proposed constraint dominance principle-based path comparison strategy and the path similarity-based multimodal solutions selection strategy, the proposed MMOEA-CDP is compared with its variants on the representative test instances in MMMSPP test suite. Compared with the proposed MMOEA-CDP, the path similarity-based multimodal solutions selection strategy is eliminated in MOEA-CDP, the constraint dominance principle-based path comparison strategy is removed in MMOEA, and both of these proposed strategies are omitted in MOEA. The parameters of these algorithms remain consistent with those described above.

The average and standard deviation of NOS values obtained by the original MMOEA-CDP and its variants on the selected test instances over thirty independent runs are presented in Table V. It is evident that the results achieved by the original MMOEA-CDP significantly outperform those of its variants. The performance of MMOEA is superior in addressing problems of type I and II, but it exhibits significant limitations when dealing with problems of type III. This can be attributed to the absence of the CDP-based path comparison strategy, which leads to its inability to handle necessary points constraints present in type III problems. Conversely, type I and II problems do not involve such constraints. Due to the absence of the path similarity-based multimodal solutions selection strategy, MOEA-CDP performs poorly on problems 5, 8-10, and 12, which involve a large number of equivalent optimal solutions. Lacking both of the proposed strategies, MOEA performs poorly on most problems. It is only able to find all equivalent optimal solutions in a few relatively simple subproblems with fewer equivalent solutions.

Fig. 11 shows the average NOS numerical curves of MMOEA-CDP and its standard deviations after running 30 times independently on the selected test instances in MMMSPP test suite. The shaded region represents the measure of standard deviation. It is evident that algorithms lacking the path similarity-based multimodal solutions selection strategy, such as MOEA and MOEA-CDP, exhibit premature convergence issues and are susceptible to premature convergence. The absence of any change in NOS values throughout the generations when solving problems with necessary point constraints, such as problem 11 and 12, in algorithms like MOEA and MMOEA that do not employ the CDP-based path comparison strategy suggests a critical need for its utilization to effectively address such problems.

C. RQ3: Discussion About the Necessary Point Constraint

The effectiveness of modeling necessary points as constraints is verified through experiments conducted on problem 12, incorporating two necessary point constraints. MMOEA-CDP serves as the optimization algorithm, with parameters consistent with previous studies. The final result is obtained by averaging 30 independent runs. Fig. 12 (a) illustrates the PF of problem 12 in the MMMSPP test suite. The cyan squares represent the true PF of the original problem 12, while the green prisms depict the PF after removing one necessary point constraint. Additionally, the red spheres indicate the PF of the unconstrained problem 12 after eliminating all necessary point constraints. The observation reveals that when the objective functions remain unchanged, removing the necessary point constraint leads to a relaxation of the satisfaction condition for the optimal solution and causes the PF to shift closer towards the coordinate origin. If the algorithm solely prioritizes convergence over feasibility, it will converge towards the red unconstrained PF and fail to accurately capture the true PF. Conversely, if feasibility is disregarded by the algorithm, it lacks convergence pressure and may prematurely halt at a local optimum that deviates significantly from the true PF. Fig. 12 (b)-(d) shows the number of equivalent Pareto optimal solutions corresponding to each point on the PF. It is evident that a single point on PF corresponds to multiple solutions in the decision space. Furthermore, the PF of problems with the same objective functions but differing in constraints exhibit overlapping regions. For instance, in the variations of problem 12, both the PF of the problems without the necessary point constraint and the problem with one necessary point exhibit optimal solutions with objective values of (41, 16.2, 18.4), (47, 14.4, 19.1), and (47, 16, 14.8).

VII. Conclusion and Future Work

In real-world traffic scenarios, we frequently encounter diverse unexpected conditions, such as abrupt disruptions in road networks and unforeseen congestion events. Merely seeking a few optimal path schemes that fulfill the objective requirements is often insufficient; instead, decision makers require a greater number of path schemes. The aim of multimodal multi-objective multi-point path planning is to identify all equivalent optimal solutions that satisfy the target requirements and desired points, thereby offering drivers and traffic decision makers an expanded range of route choices.

In this paper, a multi-objective evolutionary algorithm MMOEA-CDP is proposed to solve the multimodal multi-objective multi-point shortest path planning problem. The MMOEA-CDP algorithm possesses the capability to identify all equivalent optimal solutions in the multi-objective path planning problem, while considering necessary point constraints. A constraint dominance principle-based path comparison strategy is proposed to relax the necessary point constrains to enable the algorithm to traverse infeasible areas and discover the true constrained Pareto front. A multimodal solution selection strategy grounded in path similarity is proposed to effectively maintain the diversity of decision space solutions and preserve all equivalent Pareto optimal solutions. In the experiments, the proposed MMOEA-CDP algorithm is compared with state-of-the-art path planning algorithms in the benchmark test suite, demonstrating its performance in solving MMMSPP problems through experimental results.

In the future work, we will primarily focus on investigating large-scale path planning problems that involve multiple objectives, an increased number of necessary points, and exhibit dynamic and time series characteristics. Furthermore, we will extend the proposed MMOEA-CDP to address the traveling salesman problem (TSP), vehicle routing problem (VRP), and their respective variations. Simultaneously, we will explore the application of innovative technologies and concepts to address MMMSPP issues, encompassing novelty search and quality-diversity optimization.

References

[1] Z. Zhang, H. Qin, and Y. Li, "Multi-objective optimization for the vehicle routing problem with outsourcing and profit balancing," *IEEE Trans. Intell. Transp. Syst.*, vol. 21, no. 5, pp. 1987–2001, May 2020.

[2] S. A. Bouziaren and B. Aghezzaf, "An improved augmented ε-constraint and branch-and-cut method to solve the TSP with profits," *IEEE Trans. Intell. Transp. Syst.*, vol. 20, no. 1, pp. 195–204, Jan. 2019.

[3] L. Pasandi, M. Hooshmand, and M. Rahbar, "Modified a algorithm integrated with ant colony optimization for multi-objective route-finding; case study: Yazd," *Appl. Soft Comput.*, vol. 113, Dec. 2021, Art. no. 107877.

[4] S. Zajac and S. Huber, "Objectives and methods in multi-objective routing problems: A survey and classification scheme," *Eur. J. Oper. Res.*, vol. 290, no. 1, pp. 1–25, Apr. 2021.

[5] D. González, J. Pérez, V. Milanés, and F. Nashashibi, "A review of motion planning techniques for automated vehicles," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 4, pp. 1135–1145, Apr. 2016.

[6] F. J. Pulido, L. Mandow, and J. L. Pérez de la Cruz, "Multiobjective shortest path problems with lexicographic goal-based preferences," *Eur. J. Oper. Res.*, vol. 239, no. 1, pp. 89–101, Nov. 2014.

[7] L. Chen and J. Chung, "Mobility-aware and congestion-relieved dedicated path planning for group-based emergency guiding based on Internet of Things technologies," *IEEE Trans. Intell. Transp. Syst.*, vol. 18, no. 9, pp. 2453–2466, Sep. 2017.

[8] R. Tanabe and H. Ishibuchi, "A review of evolutionary multimodal multiobjective optimization," *IEEE Trans. Evol. Comput.*, vol. 24, no. 1, pp. 193–200, Feb. 2020.

[9] F. Kudo, T. Yoshikawa, and T. Furuhashi, "A study on analysis of design variables in Pareto solutions for conceptual design optimization problem of hybrid rocket engine," in *Proc. IEEE Congr. Evol. Comput. (CEC)*, Jun. 2011, pp. 2558–2562.

[10] S. Cheng et al., "Dynamic multimodal optimization: A preliminary study," in *Proc. IEEE Congr. Evol. Comput. (CEC)*, Jun. 2019, pp. 279–285.

[11] S. Han, K. Zhu, M. Zhou, and X. Cai, "Competition-driven multimodal multiobjective optimization and its application to feature selection for credit card fraud detection," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 52, no. 12, pp. 7845–7857, Dec. 2022.

[12] N. Tarrisson, M. Sébag, O. Teytaud, J. Lefèvre, and S. Baillet, "Multi-objective multi-modal optimization for mining spatio-temporal patterns," in *Proc. Actes de CAP 05, Conf. Francophone Sur l'Apprentissage Automatique*, Nice, France, Jan. 2005, pp. 217–230.

[13] Z. Ren, Z. B. Rubinstein, S. F. Smith, S. Rathinam, and H. Choset, "ERCA: A new approach for the resource constrained shortest path problem," *IEEE Trans. Intell. Transp. Syst.*, vol. 24, no. 12, pp. 14994–15005, Dec. 2023.

[14] L. Mandow and J.-L. Pérez-de-la Cruz, "A new approach to multiobjective a search," in *Proc. IJCAI Int. Joint Conf. Artif. Intell.*, 2005, pp. 1–6.

[15] A. Sede no-noda and M. Colebrook, "A biobjective Dijkstra algorithm," *Eur. J. Oper. Res.*, vol. 276, no. 1, pp. 106–118, Jul. 2019.

[16] P. Maristany de las Casas, A. Sedeño-Noda, and R. Borndörfer, "An improved multiobjective shortest path algorithm," *Comput. Operations Res.*, vol. 135, Nov. 2021, Art. no. 105424.

[17] C. Horoba, "Exploring the runtime of an evolutionary algorithm for the multi-objective shortest path Problem*," *Evol. Comput.*, vol. 18, no. 3, pp. 357–381, Sep. 2010.

[18] M. Nazarahari, E. Khanmirza, and S. Doostie, "Multi-objective multi-robot path planning in continuous environment using an enhanced genetic algorithm," *Exp. Syst. Appl.*, vol. 115, pp. 106–120, Jan. 2019.

[19] P. Hansen, "Bicriterion path problems," in *Multiple Criteria Decision Making Theory and Application*, vol. 177. Heidelberg, Germany: Springer, 1980, pp. 109–127.

[20] E. W. Dijkstra, "A note on two problems in connexion with graphs," in *Edsger Wybe Dijkstra: His Life, Work, and Legacy*, vol. 45, 1st ed., New York, NY, USA: ACM, 2022, pp. 287–290.

[21] E. Q. V. Martins, "On a multicriteria shortest path problem," *Eur. J. Oper. Res.*, vol. 16, no. 2, pp. 236–245, May 1984.

[22] P. E. Hart, N. J. Nilsson, and B. Raphael, "A formal basis for the heuristic determination of minimum cost paths," *IEEE Trans. Syst. Sci. Cybern.*, vol. SCS-4, no. 2, pp. 100–107, Jul. 1968.

[23] S. B. Stewart and C. C. White, "Multiobjective A," *J. ACM*, vol. 38, no. 4, pp. 775–814, Oct. 1991.

[24] P. Dasgupta, P. P. Chakrabarti, and S. C. DeSarkar, *Multiobjective Heuristic Search: An Introduction To Intelligent Search Methods for Multicriteria Optimization*. Cham, Switzerland: Springer, 1999.

[25] L. Galand and P. Perny, "Search for compromise solutions in multiobjective state space graphs," in *Proc. 17th Eur. Conf. Artif. Intell. (ECAI)*, Riva del Garda, Italy. Amsterdam, The Netherlands: IOS Press, May 2006, pp. 93–97.

[26] L. Mandow and J. L. Pérez de la Cruz, "A heuristic search algorithm with lexicographic goals," *Eng. Appl. Artif. Intell.*, vol. 14, no. 6, pp. 751–762, Dec. 2001.

[27] L. Mandow and José. L. P. De La Cruz, "Multiobjective a search with consistent heuristics," *J. ACM*, vol. 57, no. 5, p. 27, Jun. 2008.

[28] Z. Ren, R. Zhan, S. Rathinam, M. Likhachev, and H. Choset, "Enhanced multi-objective a using balanced binary search trees," in *Proc. Int. Symp. Combinat. Search*, Jul. 2022, vol. 15, no. 1, pp. 162–170.

[29] C. H. Ulloa, W. Yeoh, J. A. Baier, H. Zhang, L. Suazo, and S. Koenig, "A simple and fast bi-objective search algorithm," in *Proc. Int. Conf. Automated Planning Scheduling*, vol. 30, Jun. 2020, pp. 143–151.

[30] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Trans. Evol. Comput.*, vol. 6, no. 2, pp. 182–197, Apr. 2002.

[31] C. Wang, B. Ma, and J. Sun, "A co-evolutionary genetic algorithm with knowledge transfer for multi-objective capacitated vehicle routing problems," *Appl. Soft Comput.*, vol. 148, Nov. 2023, Art. no. 110913.

[32] Y. Wang, Y. Wei, X. Wang, Z. Wang, and H. Wang, "A clustering-based extended genetic algorithm for the multidepot vehicle routing problem with time windows and three-dimensional loading constraints," *Appl. Soft Comput.*, vol. 133, Jan. 2023, Art. no. 109922.

[33] W. Zhang, D. Yang, G. Zhang, and M. Gen, "Hybrid multiobjective evolutionary algorithm with fast sampling strategy-based global search and route sequence difference-based local search for VRPTW," *Expert Syst. Appl.*, vol. 145, May 2020, Art. no. 113151.

[34] M. Dorigo, V. Maniezzo, and A. Colorni, "Ant system: Optimization by a colony of cooperating agents," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 26, no. 1, pp. 29–41, Feb. 1996.

[35] H. Zhang, Q. Zhang, L. Ma, Z. Zhang, and Y. Liu, "A hybrid ant colony optimization algorithm for a multi-objective vehicle routing problem with flexible time windows," *Inf. Sci.*, vol. 490, pp. 166–190, Jul. 2019.

[36] M. Chica, Ó. Cordón, S. Damas, and J. Bautista, "Multiobjective constructive heuristics for the 1/3 variant of the time and space assembly line balancing problem: ACO and random greedy search," *Inf. Sci.*, vol. 180, no. 18, pp. 3465–3487, Sep. 2010.

[37] K. Deb and S. Tiwari, "Omni-optimizer: A generic evolutionary algorithm for single and multi-objective optimization," *Eur. J. Oper. Res.*, vol. 185, no. 3, pp. 1062–1087, Mar. 2008.

[38] Y. Liu, H. Ishibuchi, Y. Nojima, N. Masuyama, and K. Shang, "A double-niched evolutionary algorithm and its behavior on polygon-based problems," in *Parallel Problem Solving from Nature—PPSN*, A. Auger, C. M. Fonseca, N. Lourenço, P. Machado, L. Paquete, and D. Whitley, Eds., Cham, Switzerland: Springer, 2018, pp. 262–273.

[39] J. J. Liang, C. T. Yue, and B. Y. Qu, "Multimodal multi-objective optimization: A preliminary study," in *Proc. IEEE Congr. Evol. Comput. (CEC)*, Vancouver, BC, Canada, Jul. 2016, pp. 2454–2461.

[40] J. Zou, Q. Deng, Y. Liu, X. Yang, S. Yang, and J. Zheng, "A dynamic-niching-based Pareto domination for multimodal multiobjective optimization," *IEEE Trans. Evol. Comput.*, vol. 28, no. 5, pp. 1529–1543, Oct. 2024.

[41] C. Luo, X. Li, W. Gong, and L. Gao, "Affinity propagation hierarchical memetic algorithm for multimodal multi-objective flexible job shop scheduling with variable speed," *IEEE Trans. Evol. Comput.*, early access, Jan. 1, 2025, doi: 10.1109/TEVC.2024.3521585.

[42] X. Yao, W. Li, X. Pan, and R. Wang, "Multimodal multi-objective evolutionary algorithm for multiple path planning," *Comput. Ind. Eng.*, vol. 169, Jul. 2022, Art. no. 108145.

[43] J. Zhao et al., "Path planning based on multi-objective topological map," in *Proc. IEEE Congr. Evol. Comput. (CEC)*, Jun. 2021, pp. 1719–1726.

[44] J. Jing, L. Zhang, C. Shen, and K. Zhang, "Research on multi-modal multi-objective path planning by improved ant colony algorithm," in *Bio-Inspired Computing: Theories and Application* (Communications in Computer and Information Science), L. Pan, D. Zhao, L. Li, and J. Lin, Eds., Singapore: Springer, 2023, pp. 17–28.

[45] J. Liang, C. Yue, G. Li, B. Qu, P. N. Suganthan, and K. Yu, "Problem definitions and evaluation criteria for the cec 2021 on multimodal multiobjective path planning optimization," Dec. 2020. Accessed: Apr. 03, 2025. [Online]. Available: https://www.researchgate.net/profile/Cai-Yue-3/publication/348136850_Problem_Definitions_and_Evaluation_Criteria_for_the_CEC_2021_on_Multimodal_Multiobjective_Path_Planning_Optimization/links/5fefeb3245851553a010f854/Problem-Definitions-and-Evaluation-Criteria-for-the-CEC-2021-on-Multimodal-Multiobjective-Path-Planning-Optimization.pdf

[46] R. Wang, Z. Zhou, H. Ishibuchi, T. Liao, and T. Zhang, "Localized weighted sum method for many-objective optimization," *IEEE Trans. Evol. Comput.*, vol. 22, no. 1, pp. 3–18, Feb. 2018.

[47] P. Czyżżak and A. Jaszkiewicz, "Pareto simulated annealing—A metaheuristic technique for multiple-objective combinatorial optimization," *J. Multi-Criteria Decis. Anal.*, vol. 7, no. 1, pp. 34–47, Jan. 1998.

[48] E. Zitzler and L. Thiele, "Multiobjective evolutionary algorithms: A comparative case study and the strength Pareto approach," *IEEE Trans. Evol. Comput.*, vol. 3, no. 4, pp. 257–271, Nov. 1999.

[49] E. Zitzler, L. Thiele, M. Laumanns, C. Fonseca, and V. da Fonseca, "Performance assessment of multiobjective optimizers: An analysis and review," *IEEE Trans. Evol. Comput.*, vol. 7, no. 2, pp. 117–132, Apr. 2003.

[50] M. Li, X. Han, and X. Chu, "MOEAs are stuck in a different area at a time," in *Proc. Genetic Evol. Comput. Conf. (GECCO)*, 2023, pp. 303–311.

[51] A. Hidalgo-Paniagua, M. A. Vega-Rodríguez, and J. Ferruz, "Applying the MOVNS (multi-objective variable neighborhood search) algorithm to solve the path planning problem in mobile robotics," *Expert Syst. Appl.*, vol. 58, pp. 20–35, Oct. 2016.

[52] R. Martí, V. Campos, M. G. C. Resende, and A. Duarte, "Multiobjective GRASP with path relinking," *Eur. J. Oper. Res.*, vol. 240, no. 1, pp. 54–71, Jan. 2015.

[53] K. Zhang, C. Shen, G. G. Yen, Z. Xu, and J. He, "Two-stage double niched evolution strategy for multimodal multiobjective optimization," *IEEE Trans. Evol. Comput.*, vol. 25, no. 4, pp. 754–768, Aug. 2021.

[54] W. Li, T. Zhang, R. Wang, S. Huang, and J. Liang, "Multimodal multi-objective optimization: Comparative study of the state-of-the-art," *Swarm Evol. Comput.*, vol. 77, Mar. 2023, Art. no. 101253.

[55] A. E. Rodriguez-Fernandez, L. Schäpermeier, C. Hernández, P. Kerschke, H. Trautmann, and O. Schütze, "Finding ϵ-locally optimal solutions for multi-objective multimodal optimization," *IEEE Trans. Evol. Comput.*, early access, Sep. 11, 2024, doi: 10.1109/TEVC.2024.3458855.

[56] Z. Wei, W. Gao, M. Gong, and G. G. Yen, "A bi-objective evolutionary algorithm for multimodal multiobjective optimization," *IEEE Trans. Evol. Comput.*, vol. 28, no. 1, pp. 168–177, Feb. 2024.

[57] G. Li, W. Zhang, C. Yue, and G. G. Yen, "Clustering-based evolutionary algorithm for constrained multimodal multi-objective optimization," *Swarm Evol. Comput.*, vol. 91, Dec. 2024, Art. no. 101714.

[58] R. Chai, A. Savvaris, A. Tsourdos, Y. Xia, and S. Chai, "Solving multiobjective constrained trajectory optimization problem by an extended evolutionary algorithm," *IEEE Trans. Cybern.*, vol. 50, no. 4, pp. 1630–1643, Apr. 2020.

[59] R. Chai et al., "Real-time reentry trajectory planning of hypersonic vehicles: A two-step strategy incorporating fuzzy multiobjective transcription and deep neural network," *IEEE Trans. Ind. Electron.*, vol. 67, no. 8, pp. 6904–6915, Aug. 2020.

[60] R. Chai, A. Tsourdos, A. Savvaris, S. Chai, Y. Xia, and C. L. P. Chen, "Multiobjective overtaking maneuver planning for autonomous ground vehicles," *IEEE Trans. Cybern.*, vol. 51, no. 8, pp. 4035–4049, Apr. 2021.

[61] R. Chai, A. Tsourdos, A. Savvaris, S. Chai, Y. Xia, and C. Chen, "Multiobjective optimal parking maneuver planning of autonomous wheeled vehicles," *IEEE Trans. Ind. Electron.*, vol. 67, no. 12, pp. 10809–10821, Jan. 2020.

[62] R. Chai, A. Tsourdos, S. Chai, Y. Xia, A. Savvaris, and C. L. P. Chen, "Multiphase overtaking maneuver planning for autonomous ground vehicles via a desensitized trajectory optimization approach," *IEEE Trans. Ind. Informat.*, vol. 19, no. 1, pp. 74–87, Jan. 2023.

[63] R. Chai, H. Niu, J. Carrasco, F. Arvin, H. Yin, and B. Lennox, "Design and experimental validation of deep reinforcement learning-based fast trajectory planning and control for mobile robot in unknown environment," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 35, no. 4, pp. 5778–5792, Apr. 2024.

[64] R. Chai et al., "A two phases multiobjective trajectory optimization scheme for multi-UGVs in the sight of the first aid scenario," *IEEE Trans. Cybern.*, vol. 54, no. 9, pp. 5078–5091, Sep. 2024.

[65] R. Chai, Y. Guo, Z. Zuo, K. Chen, H.-S. Shin, and A. Tsourdos, "Cooperative motion planning and control for aerial-ground autonomous systems: Methods and applications," *Prog. Aerosp. Sci.*, vol. 146, Apr. 2024, Art. no. 101005.

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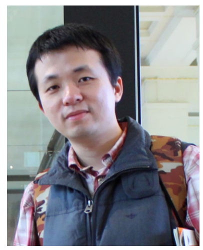

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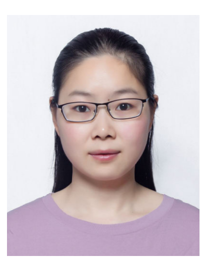

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